

Double maximal graphs of posets

Shin-ichiro Tashiro, Kenjiro Ogawa*, Morimasa Tsuchiya, Tokai University

For a finite poset $P = (V(P), \leq_P)$ and $x \in V(P)$, let $\text{Max}(P)$ be the set of all maximal elements of P , $\text{Min}(P)$ be the set of all minimal elements of P , $\text{max}(P; x) = \{m \in \text{Max}(P); x \leq_P m\}$ and $\text{min}(P; x) = \{n \in \text{Min}(P); n \leq_P x\}$. For a finite poset $P = (V(P), \leq_P)$, the *double maximal graph* of P is the graph $\text{DM}(P)$ on $V(\text{DM}(P)) = V(P)$ for which distinct vertices u and v of $\text{DM}(P)$ are adjacent if and only if $\text{max}(P; u) \subseteq \text{max}(P; v)$ and $\text{min}(P; v) \subseteq \text{min}(P; u)$, or $\text{max}(P; v) \subseteq \text{max}(P; u)$ and $\text{min}(P; u) \subseteq \text{min}(P; v)$. We give a characterization of double maximal graphs. We also show that double maximal graphs are comparability graphs. And we compare double maximal graphs with other graphs on posets, that is, upper bound graphs, double bound graphs and upper maximal graphs.

Keywords: double maximal graph, upper maximal graph, poset, upper bound graph, double bound graph