Creating Goppa codes and MDS codes from curves on cubic surfaces with twenty-seven lines over fields of odd characteristic

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A homogeneous polynomial of degree three in four variables defines a cubic surface in $\mathbf{PG}(3, \mathbb{F})$. In this case, $\mathbb{F}$ is a finite field of odd characteristic. The cubic surface $\mathcal{F}$ has twenty-seven lines, twelve of which form a Schläfli double-six. That is, a set of twelve lines partitioned into two sets of six lines $a_1, \cdots, a_6$ and $b_1, \cdots, b_6$ such that the indexing defines a bijection between the two sets. Lines in each set are pairwise skew and a_i intersects b_j when $i \neq j$. A cubic surface $\mathcal{F}$ contains forty-five tritangent planes that intersect three lines of $\mathcal{F}$. The points of $\mathcal{F}$ are mapped to points in $\mathbf{PG}(2, \mathbb{F})$ by a birational map that defines a near one-to-one correspondence with the exemption of six lines of a double-six that are mapped to six points. A (5,2)-arc determines a non-degenerate conic in $\mathbf{PG}(2, \mathbb{F})$. The zero set of the non-degenerate conic is mapped to a curve on $\mathcal{F}$ by the birational map. The points on the curve on $\mathcal{F}$ form the columns of a generator matrix for a linear code $\mathcal{C}$. The linear code $\mathcal{C}$ constructed in this manner satisfies both the Griesmer bound and Singleton bound. In certain cases, an MDS code is formed. The objective is to determine the number of distinct linear codes formed by using this method.

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