Irredundance Graphs, Part 1

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The concept of irredundance in graphs generalizes that of independence. It also provides a certificate for a dominating set in a graph to be minimal dominating. We study the upper irredundance graph (IR-graph for short) of a given graph G – the ways in which maximum irredundant sets (defined below) of G can be reconfigured successively into other such sets by exchanging (swapping) a single vertex for a neighbour in each step.
For a set $X = \{v_1, ..., v_k\}$ of vertices of a graph G, denote the closed neighbourhood of v_i (i.e., v_i and all its neighbours) by X_i and consider the collection $\mathcal{S} = \{X_1, ..., X_k\}$. The set X is irredundant if $\mathcal{S}$ has a system of distinct representatives $s_1, ..., s_k$ with the additional constraint that the representative of each set is not an element of any of the other sets. The upper irredundance number $\mathrm{IR}(G)$ is the largest cardinality of an irredundant set of G; an $\mathrm{IR}(G)$-set is an irredundant set of cardinality $\mathrm{IR}(G)$. The IR-graph of G has the $\mathrm{IR}(G)$-sets of G as vertex set, and sets X and X' are adjacent if and only if X' is obtained from X by exchanging a single vertex in X for an adjacent vertex in X'.
The main focus of this talk is the realizability of graphs as IR-graphs. In particular, all disconnected graphs are IR-graphs; that is, given a disconnected graph H as target graph, we sketch a proof that there exists a source graph G whose IR-graph is isomorphic to H. Surprisingly, though, not all connected graphs are IR-graphs.