

A Formal Construction of any Clifford Graph Algebra and Relationships Between Generators of its Different Bases

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A Clifford graph algebra $GA(G)$ is a useful structure for studying a simple graph G with n vertices. Such an algebra associates each of its n generators with one of the n vertices of G in a way that depicts the connectivity of G in that any two generators anti-commute or commute depending on whether their corresponding vertices share or do not share an edge.

In recent talks we developed these algebras for special classes of graphs by selecting a special set of generators from a basis for a classical Clifford algebra. These constructions prompt the question as to whether or not $GA(G)$ will always exist. In this talk we will prove this conjecture by modifying an idea due to A. Macdonald and construct the Clifford graph algebra for any given simple finite graph G . In our approach each monomial in the basis for $GA(G)$ is a generalized Kronecker delta defined on sequences of vectors from an orthonormal basis $\{e_1, \dots, e_n\}$ for \mathbb{R}^n .

We will explore the extent to which a different orthonormal basis $\{f_1, \dots, f_n\}$ for \mathbb{R}^n can produce generators which depict the same algebra $GA(G)$ relative to the basis $\{e_1, \dots, e_n\}$ for \mathbb{R}^n used to construct $GA(G)$.

Keywords : Clifford algebra, generalized Kronecker delta, orthonormal basis.