Decomposing intervals into triples

Aaron Meyerowitz, Florida Atlantic University

It turns out that the interval $\{0, 1, 2, \cdots, 299\}$ can be decomposed into 100 sets of the form $\{i, i+5, i+21\}$ and $\{i, i+16, i+21\}$. In other words, the interval is tiled by translates of the tile $A = \{0, 5, 21\}$ and its reflection $B = \{0, 16, 21\}$. The greedy algorithm: "work from the left and use B only when forced" gives that tiling. For any integer triple $A = \{0, a, a+b\}$ with $a < b$, this greedy algorithm leads to the tiling of an interval. We discuss the length of this interval and the structure of the tiling. For a real triple $\{0, 1, \lambda\}$, this greedy algorithm, appropriately adjusted, will yield a tiling of the non-negative reals. In the event that $\lambda = \frac{a+b}{a}$ is rational, this is by repeated tiling of finite intervals. For λ irrational this is not the case. We also discuss the shortest interval which can be tiled by an integer triple $\{0, a, a+b\}$. Those results lead to a proof that every real triple does tile a bounded interval.

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