Harmonic morphisms of graph products

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Urakawa (2000) and Baker and Norine (2009) developed the notion of harmonic morphisms of graphs with similar properties to holomorphic maps of Riemann surfaces. We give a matrix criterion for harmonic graph morphisms which allows us to translate combinatorial questions about harmonic morphisms to linear algebra questions. We illustrate its use by showing that certain maps associated to standard graph products are harmonic, and calculating their vertical and horizontal multiplicity matrices and degrees. We examine maps of NEPS graphs, such as tensor products and Cartesian products, as well as maps of lexicographic products, including *r*-blow-ups. Our results on harmonic maps of products provide a large number of examples of various types of harmonic morphisms.

Keywords: harmonic graph morphisms, graph products

Presentation will be virtual.