

Combinatorial Resultants and Circuit Polynomials in the Algebraic Rigidity Matroid

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Circuit polynomials are the unique polynomials of minimal support in a prime ideal I . Their name comes from the fact that they correspond to the circuits of the algebraic matroid induced by I . We will be interested in computing the circuit polynomials of the algebraic rigidity matroid (on n vertices) in two dimensions, i.e. the matroid induced by the Cayley-Menger ideal (on n points in 2D), CM for short. Circuit polynomials in the CM ideal are in a 1-1 correspondence with rigidity circuits in the plane, and moreover, any (real) solution of a circuit polynomial corresponds to a set of possible edge-lengths for the corresponding rigidity circuit.

The computation of circuit polynomials in the CM ideal is a non-trivial task, as they can have millions of monomial terms. Unfortunately, the general Gröbner basis methods fail for rigidity circuits with as few as 6 vertices. However, by introducing a new operation on graphs called *the combinatorial resultant*, we were able to compute the circuit polynomials of all rigidity circuits with 6 vertices, as well as a few with 7 and a few with 8 vertices.

In this talk I will describe our methods for computing circuit polynomials. This work is joint with Ileana Streinu.

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