Applications of Resistance Distance in the context of Association Schemes

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The notion of resistance distance (RD) [1, 2, 3] as a convenient metric for graphs derives from the concept of equivalent resistance for electrical circuits and has numerous applications, in particular, in organic chemistry, physics and random walks on graphs. More recently, there was proposed a procedure, called resistance-distance transform (RDT), that can be thought of as an alternative to the Weisfeiler-Leman stabilization (WLS) [4] or Jordan stabilization (JS). RDT can be applied to any simple graph Γ, as well as to a complete undirected graph with colored edges. When acting on Γ, RDT attributes to each edge (and non-edge) its corresponding RD value. Thus the outcome of this procedure constitutes a complete symmetric colored graph. Surprisingly, for many classes of experimentally investigated graphs, just one iteration of RDT provides the same result as the (symmetrized) WLS and JS [5].
At the same time, as a graph "coloring" tool, RD for general graphs is prone to numerical coincidences: two edges of distinct resistance may have the same RD and thus be mistakenly placed in the same equivalence class. Moreover, RD may result in different classification for a graph and for its complement. As an improved version of RDT which doesn't suffer from these issues, we propose using its symbolical form, similar to the onset of WLS. Specifically, each edge of the original simple graph receives one color, say red (r), and each non-edge is labeled by another color, say blue (b). Then RD for each edge is proportional to a polynomial in r and b. Based on these polynomials, edges can be places in new equivalence classes and assigned new colors. This procedure is to be repeated until stabilization. In this talk, we summarize the main features and implications of the new approach.

References

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[5] Joint results of MK with M. Klin presented at WL2018 (Pilsen, Czech Republic) and G2D2 (Yichang, China, 2019) conferences.

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