Failed positive semidefinite zero forcing for some classes of graphs

Jobby Jacob* and Yutong Wu, Rochester Institute of Technology

Consider a 2-coloring of graph G, say using colors blue and white, and let S be the set of blue vertices. The positive semidefinite zero forcing rule is as follows:

- For each component G_i of $G - S$, consider $G_i + A$, where A is the set of neighbors of the vertices in G_i from S.
- Apply zero forcing color change rule to $G_i + A$. That is, a white vertex is changed to a blue vertex if it is the only white neighbor of a blue vertex.
- Update S and repeat.

The maximum cardinality of S that fails to make all vertices of G blue while applying the positive semidefinite zero forcing rule, denoted by F^+, is called the failed positive semidefinite zero forcing number. We characterized graphs with large F^+ and graphs with small F^+. We also established F^+ for different classes of graphs, including some grid graphs.

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