Enumerating Unit Interval Parking Functions

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In 1966, Alan G. Konheim and Benjamin Weiss defined “parking functions” as follows: We have a one-way, one-lane street with n parking spaces, numbered in consecutive order from 1 to n, and we have n cars in line waiting to park. Each driver has a favorite (not necessarily distinct) parking spot, which we call its *preference*. We order these preferences in a *preference vector*. As each car parks, it drives to its preferred spot. If that spot is open, the car parks there; if not, it parks in the next available spot. If a preference vector allows all cars to park, we call it a *parking function*. In 1974, Henry O. Pollak proved the total number of parking functions of length n, meaning there are n parking spots and n cars, to be $(n+1)^{n-1}$.

In this presentation, we describe a recursive formula and explicit formula for classical parking functions, define and explicitly enumerate interval parking functions, and define unit interval parking functions (proposed by Dr. Shanise Walker, University of Wisconsin-Eau Claire) before enumerating them via a bijection with Fubini rankings. We conclude with a discussion of other parking function generalizations.

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