Prime labelings on planar grid graphs

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A graph $G$ is said to be prime if there is a bijective function $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ such that $f(u)$ and $f(v)$ are relatively prime whenever $u$ is adjacent to $v$. It is known that for any prime $p$ and any integer $n$ such that $1 \leq n \leq p$, there exists a prime labeling on the $p \times n$ planar grid graph $P_p \times P_n$. We show that $P_p \times P_n$ has a prime labeling for any odd prime $p$ and any integer $n$ such that $p < n \leq p^2$. We discuss how this approach may lead to prime labeling on $P_p \times P_n$ for any odd prime $p$ and any positive integer $n$.

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