The Wiener index: from trees to graphs with many cut-edges

Anne Christiono*, William P. Clements High School; Hua Wang, PhD, Georgia Southern University; Albert Jiang, Jasper High School; Kevin Xiao, Westwood High School

The Wiener index of a graph, defined as the sum of distances over all pairs of vertices, is one of the most well studied topological indices in chemistry and mathematics. In graph theory, properties of trees can be generalized to connected graphs with many cut-edges and multiple pseudo-components, in which a general graph can be considered as a tree with its pseudo-components as nodes. Existing algorithms for computing the Wiener index of trees—such as the Floyd-Warshall algorithm—can still be utilized to assist in computing this index in a more general graph. This generalization can also be applied in extremal graph theory, namely when finding the types of graphs that yield a maximum or minimum value of the Wiener index. The tree configuration that produces a minimum Wiener index can be generalized to describe an extremal graph with many pseudo-components. However, limitations do exist. For instance, a tree of maximum Wiener index is not so easily generalized for graphs with many cut-edges. In this talk, we showcase such generalizations and provide insights on the characteristics of graphs with many pseudo-components using characteristics of similarly structured trees.

Keywords: components, trees, cut-edge, Wiener index