$(k+1)$-line graphs of k-trees

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Let G be a k-tree of order at least $k+2$. The $(k+1)$-line graph of G, denoted by $\ell_{k+1}(G)$, is the graph whose vertices are $(k+1)$-cliques of G and two $(k+1)$-cliques are adjacent in $\ell_{k+1}(G)$ if and only if they have k vertices in common. Recently, Oliveira et al. proved that the $(k+1)$-line graph of G is a connected block graph. In this talk, we will introduce a new concept called the k-clique graph of a k-tree G, which is denoted by $G/[k]$ and defined as a graph whose vertices are k-cliques of G, and two k-cliques are adjacent in $G/[k]$ if and only if they are contained in a common $(k+1)$-clique of G. We show that $\ell_{k+1}(G)$ is isomorphic to the block graph of $G/[k]$, and $G/[k]$ is a connected block graph. By the Szeged-Wiener Theorem, the Wiener index of $\ell_{k+1}(G)$ and the Szeged index of $\ell_{k+1}(G)$ are equal, and the same property holds for $G/[k]$. A relation between the Wiener index of $G/[k]$ and the Wiener index of its block graph $\ell_{k+1}(G)$ is provided as a natural generalization of the relation between the Wiener index of a tree and the Wiener index of its line graph. The above results further develop our recent work "k-Wiener index of a k-plex" in *J. Comb. Optim.* because the Wiener index of $G/[k]$ is equivalent to the k-Wiener index of a k-tree G introduced there.

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