

Passing drops and descents

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By comparing two ways of counting all possible juggling patterns involving one juggler using at most b objects, period n , and every “beat” involves a throw, it is possible to give a combinatorial proof of Worpitzky’s identity.

A generalization of juggling patterns is *passing* patterns where there are multiple jugglers and objects are now allowed to pass between jugglers. We will show how to count the number of such patterns with k jugglers, at most b objects, and period n . As a consequence we will give a combinatorial proof for the following generalization of Worpitzky’s identity:

$$\binom{x}{k}^n = \sum_{\ell} \langle n \rangle_{\ell, k} \binom{x + \ell}{kn}$$

where $\langle n \rangle_{\ell, k}$ is the number of arrangements of $\{1^{(k)}, 2^{(k)}, \dots, n^{(k)}\}$ with ℓ descents.

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