Switch homomorphisms of (m, n)-mixed graphs

Richard Brewster* Thompson Rivers University; Arnott Kidner, Gary MacGillivray University of Victoria.

A mixed graph is a set of vertices together with an edge set and arc set (with no pair of vertices joined by both an edge and an arc). An (m, n)-mixed graph G is obtained from a mixed graph by assigning to each edge one of m colours and to each arc one of n colours. A *switch* at a vertex v of G permutes the edge colours, the arc colours, and the arc direction of edges and arcs incident to v. The group of all allowed switches is Γ.

For $k \geq 1$, the Γ-*switchable k-colouring problem* takes as input an (m, n)-mixed graph G and asks if there is a sequence of switches on vertices of G so that the resulting (m, n)-coloured graph admits a homomorphism to a target on at most k vertices. (This generalizes the notion of homomorphisms of signed graphs where $m = 2, n = 0$ and $\Gamma = S_2$.) We show the problem is polynomial time solvable for $k \leq 2$ and NP-hard for $k \geq 3$. This provides a step towards a general dichotomy theorem for the Γ-switchable homomorphism decision problem.

Keywords: graph homomorphism, edge switching, mixed graph