Extremal Decompositions for Nordhaus-Gaddum Theorems

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A Nordhaus-Gaddum theorem states bounds on $p(G) + p\left(\overline{G}\right)$ and $p(G) \cdot p\left(\overline{G}\right)$ for some graph parameter $p(G)$. We consider the sum upper bound for degeneracy, chromatic number, list chromatic number, and more. Viewing $\left\{G, \overline{G}\right\}$ as a decomposition of K_n, we describe a strategy to determine the extremal decompositions for these parameters. This produces short proofs of several existing results as well as several new theorems. We also consider generalizations to decompositions with more than two parts.

Keywords: Nordhaus-Gaddum, chromatic number, degeneracy, list coloring