

## Kernels of $(H, k)$ -reachability

Germán Benítez-Bobadilla\*, Hortensia Galeana-Sánchez, César Hernández-Cruz, Universidad Nacional Autónoma de México

Let  $H$  be a digraph, possibly with loops,  $D$  be a digraph, and  $k \geq 3$ . An  $H$ -coloring  $\zeta$ , is a map  $\zeta : A(D) \rightarrow V(H)$ . A path  $P$  in  $D$  is an  $H$ -path if the consecutive colors on  $P$  also form a walk in  $H$ . We introduce the reachability by  $(H, k)$ -paths as follows, for  $u, v \in V(D)$ , we say that  $u$  reaches  $v$  by  $(H, k)$ -paths if there exists an  $H$ -path, with length at most  $k$ , from  $u$  to  $v$  in  $D$ . Naturally, with this new reachability, the concepts of  $(H, k)$ -independent set,  $(H, k - 1)$ -absorbent set and  $(H, k)$ -kernel arises.

A digraph  $D$  is  $(H, k - 1)$ -path-quasi-transitive, if for every three vertices  $x, y$  and  $w$  of  $D$  such that there are an  $(H, k - 1)$ -path from  $x$  to  $y$  and an  $(H, k - 1)$ -path from  $y$  to  $w$  in  $D$ , then there is an  $(H, k - 1)$ -path between  $x$  and  $w$  in  $D$ . We give sufficient conditions for a  $(H, k)$ -path-quasi-transitive digraph to has an  $(H, k)$ -kernel. Also, we give sufficient conditions for a partition  $\xi$  of  $V(H)$  such that the arc set colored with the colors for every part of  $\xi$  induces an  $(H, k - 1)$ -path-quasi-transitive digraph in  $D$ , to imply the existence of an  $(H, k)$ -kernel in  $D$ .

This new reachability concept can be used to model several connectivity problems.

Keywords: Kernel by  $(H, k)$ -paths,  $(H, k)$ -reachability, kernel by  $H$ -paths.