The Threshold Strong Dimension of Starlike Trees

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Let G be a graph and W be a set of vertices of G. A vertex w in W is said to strongly resolve two vertices u and v in G if there is either a shortest u-w path that contains v or a shortest v-w path that contains u. The set W is called a strong resolving set if every pair of vertices in G is strongly resolved by a vertex of W. A smallest strong resolving set is called a strong basis and its cardinality, the strong dimension, denoted $\beta_s(G)$. One may ask: what happens to $\beta_s(G)$ if edges are added to G? It turns out that $\beta_s(G)$ could either increase, decrease or stay the same. This led to the introduction of a new parameter, the threshold strong dimension of G, denoted $\tau_s(G)$, and defined as the smallest strong dimension among all graphs having G as a spanning subgraph. Finding the threshold strong dimension, $\tau_s(T)$, of a tree T is a challenging problem. If T is a starlike tree with major vertex of degree less than 9, we show that, except for a few exceptions, $\tau_s(T)$ is 2.

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