Additive Combinatorics I: An introduction
Additive Combinatorics II: Open questions

Béla Bajnok, Gettysburg College

Additive combinatorics is one of the most exciting areas within mathematics these days. This newly-developing and rapidly-advancing field combines techniques from many different areas, and its results have attracted much recent attention across mathematics.

The field can be described as the study of combinatorial properties of sumsets (collections of sums with terms from given subsets) in additive structures. For example, given a finite subset A in an abelian group G, one can consider the sumset $A+A$ consisting of all two-term sums of the elements of A, and then ask how small this sumset may be compared to the size of A; furthermore, given that $A + A$ is small, one can examine what can be said about the structure of A.

As a rather new field within mathematics, additive combinatorics is just now coming to its own; although some of its results have been known for a very long time, many of its most fundamental questions have only been settled recently or are still unsolved. For example, the question about the minimum size of $A+A$ mentioned above was first answered in groups of prime order by Cauchy in 1813 (then re-discovered by Davenport over a hundred years later), but it was only determined in the general setting in the twenty-first century. For this and many other reasons, additive combinatorics provides an excellent area for research at many different levels: it has intriguing and promising questions for everyone.

In this two-part talk we will embark on a gentle introduction to additive combinatorics, and then discuss some of its many connections, including the author's route here from his work on spherical designs. In the second part of the talk we feature a selection of recent results and intriguing open questions. Some of these are joint work, including those with Matzke, Pach, Ruzsa, and several of the author's students.