$3'$-GDDs with 4 Groups and Block Size 5

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A group divisible design, GDD$(n, m, k; \lambda_1, \lambda_2)$, is an ordered triple (V,G,B) where V is a mn-set of symbols, G is a partition of V into m sets called groups of size n each, and B is a collection of k-subsets (called blocks) of V, such that each pair of symbols from the same group occurs in exactly λ_1 blocks and each pair of symbols from different groups occurs in exactly λ_2 blocks.
A t-(v, k, λ) design is a collection of k-subsets of a v-set V, such that every t-subset of V is in exactly λ blocks.
We define a 3-GDD, denoted by 3'-GDD$(n, 4, 5, \mu_1, \mu_2)$, with 4 groups and block size 5 by combining GDDs and t-designs where;

(i) every triple occurs exactly μ_1 times if it contains elements from at most 2 groups.

(ii) μ_2 times if it has all three elements from different groups.

In addition, we prove the necessary conditions for the existence of such GDDs and show specific instances of non-existence.