Maximal size of Sperner families with a 3-union condition.

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Let $R = \{1, 2, \ldots, r\}$ be a finite set and let $\mathcal{F}$ be a family of subsets of R with the condition: if there are two sets X and Y in $\mathcal{F}$ so that X is a subset or equal to Y then both sets are equal (Sperner condition). Consider the additional condition, that the union of any three sets in $\mathcal{F}$ is not R. It was proven, about 35 years ago, that

$$|\mathcal{F}| \leq \begin{cases} \binom{r-1}{\lfloor (r-1)/2 \rfloor} + 1 & \text{if } r \text{ is even or } r = 7 \\ \binom{r-1}{(r-1)/2} & \text{if } r \text{ is odd and } r \neq 7 \end{cases}$$

for every $r > 52$ and some $r \leq 52$, if r is even, These bounds are best possible. With new estimations and faster computers, compared to those days, we are able to prove the missing cases.

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