Some Properties of Acyclic Heaps of Pieces

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Let W be an arbitrary Coxeter group with generating set S of involutions. A reduced expression for an element $w \in W$ is a minimal length word in S that represents w. The set W_c of fully commutative elements are characterized by the property that any reduced expression for $w \in W_c$ can be obtained from any other via iterated commutations of adjacent generators. If $w \in W_c$ and $sw \notin W_c$ for some $s \in S$, then we say sw is weakly complex.

Star reducible Coxeter groups are a class of Coxeter groups whose fully commutative elements have a particularly nice property. Star reducible Coxeter groups contain the finite Coxeter groups as a subclass.

A heap is an isomorphism class of labelled posets. Each heap is equipped with a set of edges and a set of vertices. Green defined a linear map ∂_E which sends each edge of a heap E to a linear combination of vertices. If $v \in \mathrm{Im}\partial_E$, we call v a boundary vertex. If e_0 is an edge of E and $\partial_E(e_0) = v$ for a vertex v, then we call v an effective boundary vertex. If $\partial_E(e_0) = v_1 + v_2$, then v_1 and v_2 are said to be linearly equivalent. A result by Stembridge states that every fully commutative element has a unique heap.

The main result we will prove here is that in the heap of a fully commutative element in a star reducible Coxeter group, every boundary vertex is linearly equivalent to an effective boundary vertex.