

Oriented Book Embeddings

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A graph G has a k -page book embedding if G can be embedded into a k -page book. The minimum k such that G has a k -page book embedding is the *book thickness* of G , denoted $bt(G)$. Most of the work on this subject has been done for unoriented graphs and oriented acyclic graphs (no directed cycles). In this work we discuss oriented graphs \vec{D} containing directed cycles by using *oriented book embeddings* and *oriented book thickness*, $obt(\vec{D})$. To characterize \vec{D} such that $obt(\vec{D}) = k$, we define the class \mathcal{M}^k of k -page critical oriented graphs to be all oriented graphs \vec{D} with $obt(\vec{D}) = k$, but for every proper oriented subgraph of \vec{D} , denoted \vec{D}' , we have that $obt(\vec{D}') < k$. Determining \mathcal{M}^k for general k is challenging; we narrow down the list of oriented graphs in \mathcal{M}^k for small k . In this work we show complete lists for \mathcal{M}^1 and for $\mathcal{M}^2 \cap \mathcal{U}$, where \mathcal{U} consists of all *strictly dicyclic oriented graphs*, that is, oriented graphs containing exactly one oriented cycle, which is a directed cycle.

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