A New Generating Function for Counting Zeroes of Polynomials

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We want to count the zeroes of a multivariate polynomial $f(x_1, \ldots, x_n)$ over a finite field F and over the associated rings of Witt vectors of finite length. It is helpful to think of the polynomial f as having coefficients in a p-adic field K whose residue field is F: then we count zeroes of f modulo each power of the unique maximal ideal of the valuation ring of K. One example would be counting zeroes of a polynomial whose coefficients are p-adic integers (possibly rational integers) modulo various powers of p. The Igusa local zeta function Z_f is a generating function that organizes all these zero counts, and its poles tell us about the p-divisibility of the counts. We devise a new method for calculating the Igusa local zeta function that involves a new kind of generating function G_f. Our new generating function is a projective limit of a family of generating functions, and contains more data than the Igusa local zeta function. The new generating function G_f resides in an algebra whose structure is naturally compatible with operations on the underlying polynomials, thus facilitating calculation of local zeta functions and helping us find zero counts of polynomials over finite fields and rings.

Keywords: polynomial, zero count, finite field, p-adic field, Igusa zeta function