An Extremal Problem Concerning Vertex Covers

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The vertex cover number b(G) of a graph G is the smallest size of a set of vertices of G which contains a representative of each edge. If b(G) = s and b(H) < s for every proper subgraph H of G, we will call G s-critical (with respect to b). Let f(s) be the maximum order of an s-critical graph. It can be shown that f(s) < s2 + 2. We conjecture that f(s) = 2s, and that the unique s-critical graph with that number of vertices is a matching with s edges, for each s. Whether this is true or not, the determination of f(s) will bear on some cases of the “strong (n,k,t) conjecture”. This conjecture boils down to: if n > k > t > 2, and G has the minimum number of edges among graphs on n vertices with the property that every induced subgraph of order k contains a clique on t vertices, then every component of G is a clique.