**Stability of the potential function**

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A graphic sequence $\pi$ is *potentially $H$-graphic* if there is some realization of $\pi$ that contains $H$ as a subgraph. The Erdős-Jacobson-Lehel problem [Graphs realizing the same degree sequence and their respective clique numbers. *Graph Theory, Combinatorics and Applications* (eds. Alavi, Chartrand, Oellerman and Schwenk), Vol. 1, 1991, 439–449] asks to determine $\sigma(H, n)$, the minimum even integer such that any $n$-term graphic sequence $\pi$ with sum at least $\sigma(H, n)$ is potentially $H$-graphic. The parameter $\sigma(H, n)$ is known as the *potential function* of $H$, and can be viewed as a degree sequence variant of the classical extremal function $\text{ex}(n, H)$. Recently, Ferrara, LeSaulnier, Moffatt and Wenger [On the sum necessary to ensure that a degree sequence is potentially $H$-graphic, to appear in *Combinatorica*] determined $\sigma(H, n)$ asymptotically for all $H$, which is a potential function analogue to the Erdős-Stone-Simonovits Theorem.

In this talk, we investigate a stability concept for the potential number, inspired by Simonovits' [A method for solving extremal problems in graph theory, stability problems. In: *Theory of Graphs, Proc. Colloq., Tihany*, 1966, Academic Press, New York, 1968, pp. 279–319] classical result on the stability of the extremal function. Simonovits' theorem demonstrates that for every nonbipartite graph $H$ with chromatic number $r + 1$, any $H$ free graph with close to $\text{ex}(n, H)$ edges must have structure approximated by that of the Turán graph $T_{n,r}$. We first define a notion of stability for the potential number that is a natural analogue to the stability given by Simonovits, and demonstrate that not all graphs are $\sigma$-stable. We then give a sufficient condition for a graph $H$ to be stable with respect to the potential function, and characterize the stability of those graphs $H$ that contain an induced subgraph of order $\alpha(H) + 1$ with exactly one edge.

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