On the Roots of Tribonacci-type Polynomials

Alexander Durbin*, Huei Sears, Michigan State University

Consider Tribonacci-type polynomials defined by the following recurrence relation $T_n(x) = \alpha(x) \cdot T_{n1}(x) + \beta(x) \cdot T_{n2}(x) + \gamma(x) \cdot T_{n3}(x)$, where coefficients $\alpha(x), \beta(x), \gamma(x)$ and initial conditions $T_0(x), T_1(x)$, and $T_2(x)$ are arbitrary functions. In this paper, we present matrix representations of $T_n(x)$, namely $M_n(x)$, such that $\det M_n(x) = T_{n-1}(x)$. Using this determinant representation, we discuss the nature of all roots of all polynomial sequences of this form using an alternative method of Gerŝchgorin's Circle Theorem, Laguerre's application of Samuelson's Inequality, and an application of Rouché's Theorem. Special cases of $T_n(x)$ include recurrences with only real roots and only complex roots. This paper is concluded with a presentation of ordinary generating functions for polynomials mentioned and the general case.

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