Hamiltonian circuits in Cayley digraphs on cyclic groups

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We consider Hamiltonian circuits in Cayley digraphs on a cyclic group with a generating set containing an element of order two. In particular, we provide results toward proving the following conjecture.

Conjecture. Let k be a positive integer and let a be an integer such that $1 \leq a \leq k$. Then $\mathrm{Cay}(Z_{2k}; a, a+1, k)$ has a hamiltonian circuit iff either k is not divisible by 6 or $a \neq k-3$.

We consider torus knot calculations of various circuits in $\mathrm{Cay}(Z_{2k}; a, a+1, k)$ in order to find Hamitonian circuits. For example, we establish the following results.

Theorem 1. Suppose a_0 and k are positive integers such that $\gcd(a_0, 2k) = 1$. Let $a = 2a_0$. Let q_1 and q_2 be integers such that $a_0 q_1 - 2k q_2 = 1$. Then $\mathrm{Cay}(Z_{4k}; a, a+1, 2k)$ has a Hamiltonian circuit.

Theorem 2. Suppose a and k are positive integers such that a is odd and a divides k. Then $\mathrm{Cay}(Z_{4k}; a, a+1, 2k)$ has a Hamiltonian circuit.

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