On Extending Hansel's Theorem to Complete Hypergraphs

Gregory Churchill*, Brendan Nagle, University of South Florida

For integers $2 \leq d \leq k \leq n$, a d-covering of $\binom{[n]}{k}$ is a family $\mathcal{D}$ of d-partite k-graphs D where every k-tuple of $\binom{[n]}{k}$ belongs to some $D \in \mathcal{D}$. Let $f(n,k,d)$ denote the minimum of $\sum_{D \in \mathcal{D}} |V(D)|$ over all d-coverings of $\binom{[n]}{k}$. When $d = k = 2$, a classical theorem of Hansel says that $\lceil n \log_2 n \rceil \leq f(n,2,2) \leq n \lceil \log_2 n \rceil$.

In this talk, we will prove the general lower bound $f(n,k,d) \geq n \log_{d/d-1}(n/(k-1))$. We highlight a construction which shows that this bound is sharp when $d = 2$, k is arbitrary, and $n/(k-1)$ is a power of two. This bound is also asymptotically sharp in a certain range of $d << k << n$, as shown by a standard probabilistic argument, which we discuss if time permits.

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