Singly perturbed non-local diffusion systems applied to disease models

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We study a model, suitable for vector-borne diseases, where we assume that the human hosts epidemiology acts on a much slower time scale than the one of mosquitoes transmitting as a vector from human to human, due to their vastly different life cycles. This particular model also includes the spatial movement of both vectors and humans, getting a system of non-local and local spatial dynamics. In a convenient setting, we prove convergence of solutions. (Joined work with M. Pereira)

The model proposed takes the following form, where $i$ and $j$ will model the density of infected human and vector population.

\begin{equation}
\begin{cases}
\frac{\partial i}{\partial t} = \alpha_h (1 - i)j - \beta_h i + d_1 K_j i, \\
\frac{\partial j}{\partial t} = \frac{\alpha_v}{\varepsilon} (1 - j)i - \frac{\beta_v}{\varepsilon} j + d_2 \Delta j,
\end{cases} \quad x \in \Omega, \ t > 0. \tag{1}
\end{equation}

We work in a regular bounded domain $\Omega \subset \mathbb{R}^n$ with exterior unit normal $N$. Also, we take the homogeneous Neumann boundary condition to the function $j$

\begin{equation}
\frac{\partial j}{\partial N} = 0, \quad x \in \partial \Omega. \tag{2}
\end{equation}

The constants $\alpha_h, \alpha_v, \beta_h, \beta_v, d_1$ and $d_2$ are positive, $\Delta$ denotes the Laplacian differential operator and $K_j$ is the following nonlocal operator

\[ K_j i(x) = \int_{\Omega} J(x - y)(i(y) - i(x))dy, \quad x \in \Omega. \]
We assume that the kernel $J$ satisfies the hypotheses

\[
J \in C(\mathbb{R}^n, \mathbb{R}) \text{ is non-negative with } \\
\begin{align*}
(\text{H}_J) \\
&J(0) > 0, \ J(-x) = J(x) \text{ for every } x \in \mathbb{R}^n, \text{ and } \\
&\int_{\mathbb{R}^n} J(x) \, dx = 1.
\end{align*}
\]

Under these conditions, the $K_J$ is known as a nonlocal operator with non-singular kernel and Neumann condition.

Although the simple toy model (1) is useful to show the difficulties when one deals with non-local operators (see [1]) coupled with the laplacian, other versions using discrete networks, ODEs or fractional Laplacians have been used in applications (see [2] [3] [4] [5]).

References


