MAC 1105 REVIEW 4 ANSWERS

Answers are indicated by capital letters or are explicitly given.

1. For the function $f(x) = \begin{cases} -(x+3)^2 & \text{if } x < -4 \\ -2x+1 & \text{if } -4 < x < -1 \\ \sqrt{x+4}-1 & \text{if } -1 \leq x \leq 5 \end{cases}$

(a) Find $f(-4)$, $f(0)$, $f(3)$.

(b) Graph $f(x)$.

(c) Find the domain of f.

(d) Find where f is increasing and decreasing.

(e) Find where f is positive and negative.

$f(-4) = \text{undefined}$

$f(0) = 1$

$f(3) = \sqrt{7} - 1$

domain: $x \in (-\infty, -4) \cup (-4, 5]$

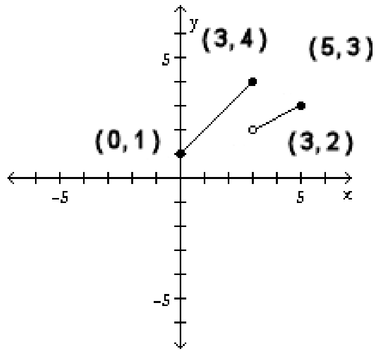
increasing: $x \in (-\infty, -4), x \in (-1, 5)$

decreasing: $x \in (-4, -1)$

positive: $x \in (-1, 5]$

negative: $x \in (-\infty, -4)$

2. For the graph of f below, define f piecewise.



(a) $f(x) = \begin{cases} x+1 & \text{if } 0 \le x < 3 \\ \frac{1}{2}x + \frac{1}{2} & \text{if } 3 \le x \le 5 \end{cases}$

(b) $f(x) = \begin{cases} x+1 & \text{if } 0 \le x < 3 \\ \frac{1}{2}x & \text{if } 3 \le x \le 5 \end{cases}$

(c) $f(x) = \begin{cases} x+1 & \text{if } 0 \le x < 3 \\ \frac{1}{2}x - \frac{1}{2} & \text{if } 3 \le x \le 5 \end{cases}$

(d) $f(x) = \begin{cases} x+1 & \text{if } 0 \le x < 3 \\ \frac{1}{2}x + 2 & \text{if } 3 \le x \le 5 \end{cases}$

(E) $f(x) = \begin{cases} x+1 & \text{if } 0 \le x \le 3 \\ \frac{1}{2}x + \frac{1}{2} & \text{if } 3 < x \le 5 \end{cases}$

3. Let $g(x) = \dfrac{2}{x+1}$. Then $\dfrac{g(x) - g(2)}{x-2}$ is equal to

(a) $\dfrac{-2x+8}{3(x+1)(x-2)}$, $x \neq 2$

(b) $\dfrac{2}{5(x+3)}$, $x \neq 2$

(c) $\dfrac{-1}{x+3}$, $x \neq 2$

(d) $\dfrac{-2x+8}{3(x+1)}$, $x \neq 2$

(E) $\dfrac{-2}{3(x+1)}$, $x \neq 2$

4. Let $f(x) = \dfrac{x-3}{x+3}$. After performing algebraic operations and simplifying the difference quotient

$$\frac{f(x+h)-f(x)}{h} =$$

(a) $\dfrac{6+h}{(x+h+3)(x+3)}$

(b) $\dfrac{-6}{x(x+3)}$

(c) $\dfrac{7}{(x+3)(x-3)}$

(D) $\dfrac{6}{(x+3)(x+3+h)}$

(e) $\dfrac{6h}{(x+3)(x+3+h)}$

5. Let $g(x) = \dfrac{-2}{x+1}$. Then $\dfrac{g(x)-g(2)}{x-2}$ is equal to

(a) $\dfrac{-2x+8}{3(x+1)(x-2)}$, $x \neq 2$

(B) $\dfrac{2}{3(x+1)}$, $x \neq 2$

(c) $\dfrac{-1}{x+3}$, $x \neq 2$

(d) $\dfrac{-2x+8}{3(x+1)}$, $x \neq 2$

(e) $\dfrac{-2}{3(x+1)}$, $x \neq 2$

6. Let $z(x) = -x^2 + 3x$. Then $\dfrac{z(x)-z(1)}{x-1}$ is equal to

(a) $1-x$, $x \neq 1$

(b) $x-2$, $x \neq 1$

(C) $2-x$, $x \neq 1$

(d) $-x+1$

(e) $-1-x$, $x \neq 1$

Questions 7 and 8 refer to $f(x) = \sqrt{x+3}$ **and** $a = 4$.

7. Geometrically, the diffence quotient $\dfrac{f(x)-f(a)}{x-a}$ is equal to

(A) the slope of the secant line passing through the point $(x, f(x))$ and the point $(4, \sqrt{7})$

(b) the slope of the tangent line passing through the point $(x, f(x))$ and the point $(4, \sqrt{7})$

(c) the slope of the secant line passing through the point $(-x, f(x))$ and the point $(4, \sqrt{7})$

(d) the average height of the function between the point $(x, f(x))$ and the point $(4, \sqrt{7})$

(e) none of the other choices

8. $\dfrac{f(x)-f(a)}{x-a}$ is equal to

(A) $\dfrac{1}{\sqrt{x+3}+\sqrt{7}}$, $x \neq 4$

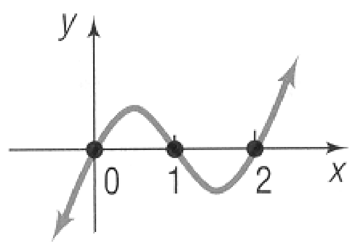
(b) $\dfrac{\sqrt{7}}{4}$

(c) $-\dfrac{\sqrt{7}}{4}$

(d) $\dfrac{1}{x-4}$

(e) $\dfrac{1}{\sqrt{x-3}+\sqrt{7}}$, $x \neq 4$

9. Choose the selection that best completes the statement: The degree of the polynomial function with graph given below is even/odd and at most/least degree ______________



(a) even, at most, 3
(B) odd, at least, 3
(c) odd, at most, 3
(d) even, at least, 4
(e) none of the other choices

10. The quadratic function $f(x) = \frac{1}{4}x^2 - x + 5$ has vertex
(A) (2,4)
(b) (-2,8)
(c) (4,5)
(d) (-4,13)
(e) none of the other choices

11. The quadratic function $f(x) = \frac{1}{4}x^2 - x + 5$ has standard form
(a) $f(x) = \frac{1}{2}(x-2)^2 - 8$
(b) $f(x) = \frac{1}{4}(x+2)^2 + 8$
(c) $f(x) = \frac{1}{2}(x-2)^2 + 8$
(d) $f(x) = \frac{1}{4}(x-4)^2 + 2$
(E) $f(x) = \frac{1}{4}(x-2)^2 + 4$

12. The quadratic function $f(x) = 2x^2 + 3x - 1$ has x-intercept(s)

(a) $(\frac{-3-\sqrt{17}}{2}, 0), (\frac{-3+\sqrt{17}}{2}, 0)$

(b) $(\frac{3-\sqrt{17}}{4}, 0), (\frac{3+\sqrt{17}}{4}, 0)$

(c) $(-1, 0), (-\frac{1}{4}, 0)$

(D) $(\frac{-3-\sqrt{17}}{4}, 0), (\frac{-3+\sqrt{17}}{4}, 0)$

(e) $(1, 0), (\frac{1}{4}, 0)$

Questions 13-18 refer to the function $h(x) = (x+4)(x+2)(x-2)^2$

13. The function $h(x)$ has x-intercept(s)

(a) $(-2, 0)$, $(-1, 0)$, $(0, 0)$
(B) $(2, 0)$, $(-2, 0)$, $(-4, 0)$
(c) $(-1, 0)$
(d) $(0, 0)$
(e) none of the other choices

14. The function $h(x)$ has y-intercept(s)

(a) $(0, -4)$
(b) $(0, 1)$
(c) $(0, 5)$
(d) $(0, 0)$, $(0, 1)$, $(0, -4)$
(E) none of the other choices

15. The domain of $h(x)$ consists of all

(A) $x \in (-\infty, +\infty)$
(b) $x \in (-2, 3)$
(c) $x \in (-\infty, -2) \cup [3, +\infty)$
(d) $x \in (-\infty, -2) \cup (-2, +\infty)$
(e) none of the other choices

16. The function $h(x)$ is negative in the interval

(a) $x \in (-\infty, -4) \cup (-2, 2) \cup (2, +\infty)$
(B) $x \in (-4 - 2)$
(c) $x \in (-\infty, +\infty)$
(d) $x \in (-2, 2) \cup (2, +\infty)$
(e) none of the other choices

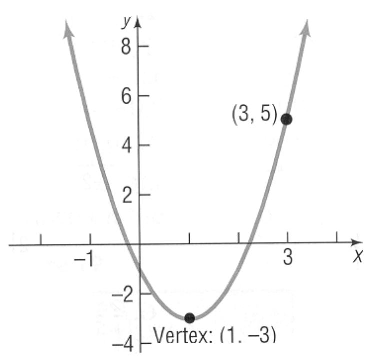
17. The function $h(x)$ is positive in the interval

(a) $x \in (-\infty, -4)$
(B) $x \in (-\infty, -4) \cup (-2, 2) \cup (2, +\infty)$
(c) $x \in (-\infty, +\infty)$
(d) $x \in (-4, -2)$
(e) none of the other choices

18. Which of the following best describes the end behaviour of $h(x)$

(a) as $x \to -\infty$, $h(x) \to +\infty$ and as $x \to +\infty$, $h(x) \to -\infty$
(b) as $x \to -\infty$, $h(x) \to -\infty$ and as $x \to +\infty$, $h(x) \to +\infty$
(c) as $x \to -\infty$, $h(x) \to -\infty$ and as $x \to +\infty$, $h(x) \to -\infty$
(D) as $x \to -\infty$, $h(x) \to +\infty$ and as $x \to +\infty$, $h(x) \to +\infty$
(e) none of the other choices

19. Find the quadratic function with graph given below.

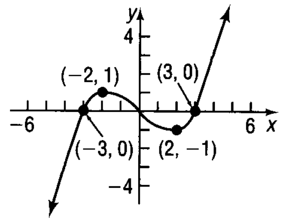


(a) $f(x) = -\frac{1}{2}(x-1)^2 - 3$
(b) $f(x) = \frac{1}{2}(x-1)^2 - 3$
(C) $f(x) = 2(x-1)^2 - 3$
(d) $f(x) = 2(x+1)^2 - 3$
(e) $f(x) = \frac{1}{2}(x-1)^2 + 3$

20. A developer wants to enclose a rectangular grassy lot that borders a city street for parking. If the developer has 200 feet fencing and does not fence the side along the street, what is the largest area that can be enclosed?

(a) 10,000 sq. ft.
(b) 7,500 sq. ft.
(c) 2,500 sq. ft.
(D) 5,000 sq. ft.
(e) 40,000 sq. ft.

21. Find the **cubic** function with graph given below.



(a) $f(x) = -2x(x+3)(x-3)$

(B) $f(x) = \frac{x^3}{10} - \frac{9}{10}x$

(c) $f(x) = 10x(x+3)(x-3)$

(d) $f(x) = 2(x^3 - 9x)$

(e) $f(x) = \frac{1}{10}(x+3)(x-3)$

Questions 22-23 refer to the following: The price p dollars and the quantitiy x sold of a certain product obey the demand equation

$$x = -15p + 450, \qquad 0 \leq p \leq 30$$

22. Express the revenue R as a function of x.

(A) $R(x) = x\left(\frac{x-450}{-15}\right)$

(b) $R(x) = -x^2 + 450x - 15$

(c) $R(x) = x^2 - 450x + 15$

(d) $R(x) = -15x^2(x - 450)$

(e) $R(x) = -15x(x - 450)$

23. What quantity x_{max} maximizes the revenue?

(a) $x_{max} = 189$

(b) $x_{max} = 200$

(C) $x_{max} = 225$

(d) $x_{max} = 450$

(e) $x_{max} = 300$

24. For the function $f(x) = -4x^2 - 6x + 5$

(a) Find the standard form of $f(x)$.

(b) Graph $f(x)$ labeling the vertex, x-intercept(s), y-intercept, and axis of symmetry.

(c) Find where f is increasing and decreasing.

(d) Find where f is positive and negative.

increasing: $x \in (-\infty, -\frac{3}{4})$

decreasing: $x \in (-\frac{3}{4}, +\infty)$

positive: $x \in (\frac{-3-\sqrt{29}}{4}, \frac{-3+\sqrt{29}}{4})$

negative: $x \in (-\infty, \frac{-3-\sqrt{29}}{4}) \cup (\frac{-3+\sqrt{29}}{4}, +\infty)$

standard form: $f(x) = -4(x + \frac{3}{4})^2 + \frac{29}{4}$