Spring 2002 Contest Problems

All problems require a solution or proof. See Competition Rules and Submission Method at our home page for details

January Contest Problems (Level 1)–Spring 2002

A1. Find the largest integer n such that $n^3 + 5$ is divisible by $n + 5$.

A2. What is the radius of the sphere that can be inscribed in the regular tetrahedron whose edges have length 1?

A3. Which of the two numbers 33^{12} and 63^{10} is larger? Prove your statement.

A4. Given a unit square and five points in it. Prove that among the five points there exist two whose distance does not exceed $1/\sqrt{2}$.

A5. Let $x_1, x_2, \cdots, x_{2002}$ be the 2002 solutions of the equation $x^{2002} + 5x - 1 = 0$. Find the sum

$$\sum_{i=1}^{2002} x_i^{2002}.$$

January Contest Problems (Level 2)–Spring 2002

B1. Given n distinct positive integers $a_1, a_2, \cdots, a_n$, where $n \geq 2$. Prove that

$$\sum_{i=1}^{n} \left(\frac{1}{a_i}\right)^2.$$

is not an integer.

B2. Let $a_1 = 2$ and for $n \geq 1$ define

$$a_{n+1} = \frac{a_n(a_n + 1)}{2}.$$

Find the last two digits of a_{2002}.

B3. Let ΔABC be the triangle in the coordinate plane with $A = (0, 0)$, $B = (6, 0)$, and $C = (2, 4)$. Let $P = (a, b)$ be any interior point of ΔABC. From P draw the perpendiculars PX, PY, and PZ to the three sides of the triangle, where X, Y, and Z are on AB, BC, and CA respectively. Then construct three squares exterior to the triangle ΔABC using AX, BY, and CZ as the sides respectively. Find the sum of the areas of these three squares.