

FAU/Stuyvesant Alumni Mathematics Competition

Fall Semester, 2000

September (First Round of Three)

Please see Competition Rules and Submission Method
on Home Page for details

Level 1 Problems

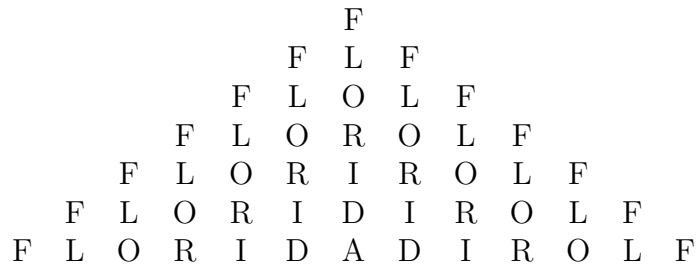
- A1.** The number $2^{48} - 1$ is divisible by two numbers between 50 and 100. What is the sum of those two numbers?
- A2.** Of 1000 couples there are 800 cases in which the husband is taller than the wife, 700 cases where the husband is heavier than the wife, and 660 cases where the husband is both taller and heavier than the wife. In how many ways will the wife be at least as tall and as heavy as the husband?
- A3.** Trapezoid $PQRS$ is circumscribed around circle O , with $PQ \parallel SR$ and $\angle PSR = \angle QRS = 45^\circ$. If $PQ = 1$, find QR .
- A4.** If the letters x and y represent single digits, find the value of x if

$$(3 \times 23x)^2 = 492y04.$$

[Note: Here, $23x$ is a 3-digit number, and $492y04$ a 6-digit number.]

- A5.** Angle B of triangle ABD is 90° . On AD there is a point C for which $AC = CD$ and $AB = BC$. Find the magnitude of angle BAD .
- A6.** Tennis balls are usually supplied in a cylindrical container which contains 3 balls in a neat fit (the centers of the balls are collinear). What proportion of the volume of the container is occupied by the three balls?
- A7.** When 1166, 1558, and 2244 are each divided by x , the remainder is the same in each case. What is the largest possible value of x ?
- A8.** If $\sec x + \tan x = 5$, find the value of $\csc x + \cot x$.

- A9.** The word FLORIDA may be spelled out in the following diagram beginning at any F and moving in either a horizontal or vertical direction to an L, and then an O, etc. How many paths may be taken to spell out the word FLORIDA?



- A10.** Mama's restaurant has two dishwashing machines, one which washes only dishes, another which washes only glasses. The first machine holds 225 dishes and takes 10 minutes longer to wash a load than the other machine which holds 75 glasses. Twice as many dishes as glasses are washed each hour.

From 12 noon, when the restaurant opens, until 9 pm., when it closes, 600 dishes and 260 glasses are used during each hour.

The two washing machines start working 30 minutes after the restaurant opens, and they operate continuously until all the glasses and dishes are washed, at which time Mama goes home.

When does Mama leave her restaurant?

Level 2 Problems

- B1.** The top and bottom sections of a frustum have areas A and B . Find the area of the section half way between top and bottom.
- B2.** Let A be a fixed point on a circle, and PQ a variable diameter. Find the locus of the perpendicular foot of A on PQ . Prove your answer.
- B3.** Find all integer solutions to the equations

$$x + y + z = 3, \quad x^3 + y^3 + z^3 = 3.$$

- B4.** Find all integers n for which

$$2^{2003} + 2^{2000} + 2^n$$

is the square of an integer.