

FAU/Stuyvesant Alumni Mathematics Competition

Fall Semester, 2000

November (Third Round of Three)

Please see Competition Rules and Submission Method
on Home Page for details

Level 1 Problems

- A21.** When a right triangle is rotated about one leg, the volume of the cone produced is 800π . When rotated about the other leg, the volume is 1920π . What is the length of the hypotenuse of the right triangle?
- A22.** If x , y , and z all exceed 1 and $w > 0$, such that $\log_x w = 24$, $\log_y w = 40$, and $\log_{xyz} w = 12$, find $\log_z w$.
- A23.** The sum of 28 consecutive positive odd integers is a perfect cube. Find the first number in the smallest such set of 28 numbers.
- A24.** In triangle ABC , $\sin A : \sin B : \sin C = 4 : 5 : 6$, while $\cos A : \cos B : \cos C = x : y : 2$. Find the value of x .
- A25.** What is the units digit of

$$1^1 + 2^2 + 3^3 + 4^4 + 5^5 + \cdots + 1999^{1999} + 2000^{2000}?$$

- A26.** Three semi-circles are drawn inside an equilateral triangle each using a side of the triangle as diameter. If the side of the triangle is 4, find the area common to all three semi-circles.
- A27.** What is the area of the larger region enclosed by the graphs of $y^2 = 2x - x^2$, $y = x - 1$, and $y = 0$?
- A28.** The vertex angle A of triangle ABC is 20° . Points D and E are selected on sides AB and AC so that $\angle CBE = 50^\circ$ and $\angle BCD = 60^\circ$. Find the value of $\angle AED$.
- A29.** If $A = 15 + \sqrt{220}$, find the units digit in the decimal expansion of $A^{19} + A^{99}$.

- A30.** Points M and N are taken on sides AD and AB of square $ABCD$ such that triangle CMN is an equilateral triangle. If the area of square $ABCD$ is 1, what is the area of triangle CMN ?

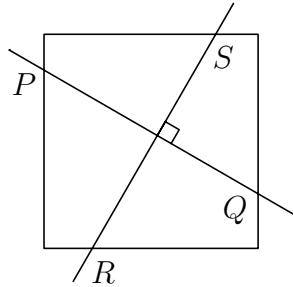
Level 2 Problems

- B9.** Find the remainder of the division of the polynomial

$$x^{81} + x^{49} + x^{25} + x^9 + x$$

by the polynomial $x^3 - x$.

- B10.** Prove that if PQ , RS are any two perpendicular lines cutting the pairs of opposite sides of a square at P , Q and R , S respectively, then $PQ = RS$.



- B11.** In the number

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choose the two missing digits in such a way that the result is divisible by 13. Find all possible solutions.

- B12.** Let $f(x) = \frac{4^x}{4^x + 2}$. Find the sum

$$f\left(\frac{1}{1001}\right) + f\left(\frac{2}{1001}\right) + f\left(\frac{3}{1001}\right) + \cdots + f\left(\frac{1000}{1001}\right)$$