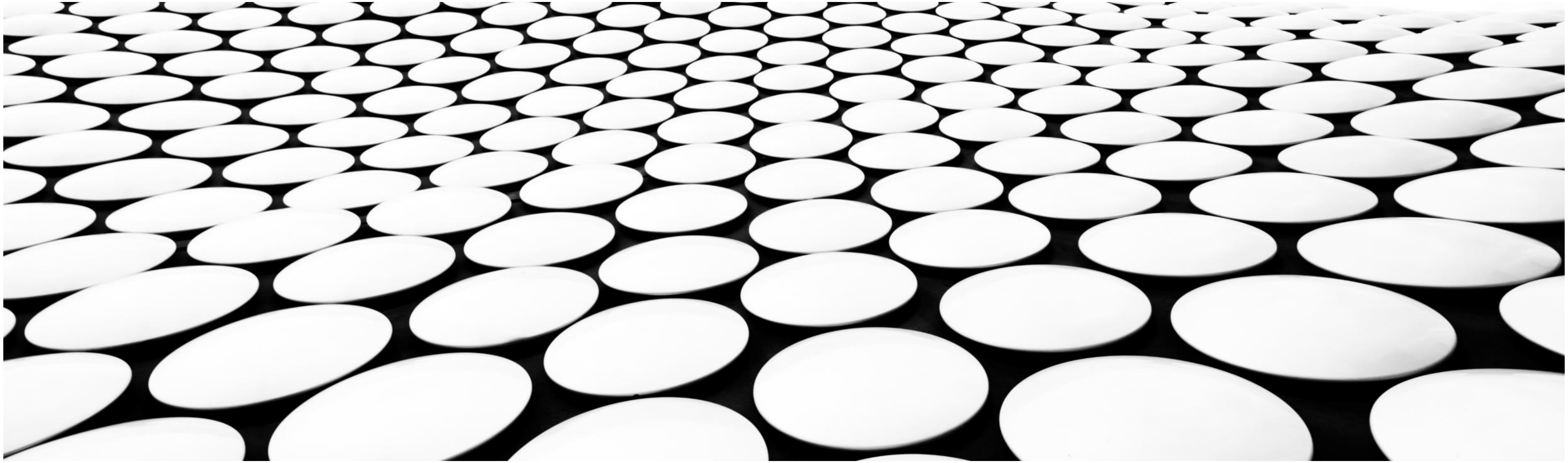
MATH CIRCLE AT FAU

3/30/2024



THE ISLAND OF KNIGHTS AND KNAVES

Here we are again on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

A world-famous logician arrives at the island on the way to Logic Hotel, the resort were everything is logically perfect. He gets to a fork on the road, he knows that one of the two paths lead to the hotel; the other one to the crocodile pit. An islander stands by the road; the logician is sure the islander knows which is the right branch. The logician thinks for a moment and then, pointing at one of the branches, asks one question. From the reply he knows which road to take. What is the question?
(From Martin Gardner's *Hexaexagons and Other Mathematical Diversions*.)

THE ISLAND OF KNIGHTS AND KNAVES

A world-famous logician arrives at the island on the way to Logic Hotel, the resort were everything is logically perfect. He gets to a fork on the road, he knows that one of the two paths lead to the hotel; the other one to the crocodile pit. An islander stands by the road; the logician is sure the islander knows which is the right branch. The logician thinks for a moment and then, pointing at one of the branches, asks one question. From the reply he knows which road to take. What is the question?

There are several possible answers. Here are two:

If I ask you `is this the road to the hotel?' will you answer yes?

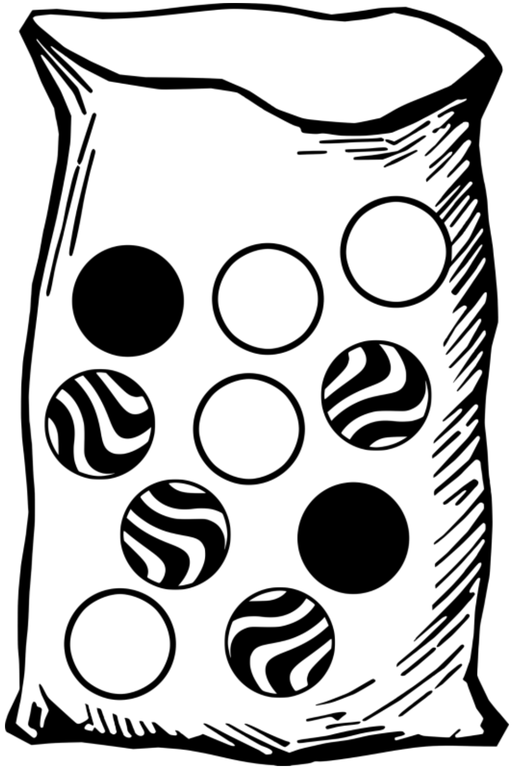
(Is this the road to the hotel and are you a knight) or (is this the wrong road and are you a knave)

LOSING ONE'S MARBLES

Suppose you have three boxes, one containing two black marbles, one containing two white marbles, and a third containing a black and a white marble.

The boxes were labeled for their contents: BB, WW, BW, but someone has switched the labels so that now every box is incorrectly labeled.

You are allowed to take one marble at a time out of any box, without looking inside, and by this process of sampling you are to determine the contents of all three boxes. What is the smallest number of drawings needed to do this? (From *Hexaexagons and Other MathematicalDiversions,* by Martin Gardner)



LOSING ONE'S MARBLES

Suppose you have three boxes, one containing two black marbles, one containing two white marbles, and a third containing a black and a white marble.
The boxes were labeled for their contents: BB, WW, BW, but someone has switched the labels so that now every box is incorrectly labeled.
You are allowed to take one marble at a time out of any box, without looking inside, and by this process of sampling you are to determine the contents of all three boxes. What is the smallest number of drawings needed to do this? (From *Hexaexagons and Other MathematicalDiversions,* by Martin Gardner)

SOLUTION: One.

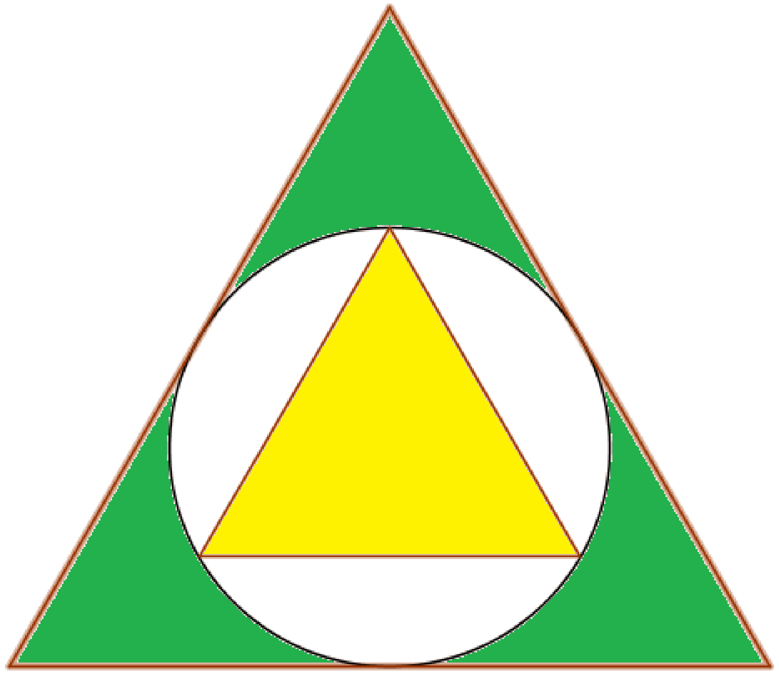
Take a marble from the BW box. Suppose it is black. Then (since the box is mislabeled), the other marble is also black, and you have the BB box. Then the WW must be the BW, the BW must be the WW box.
Similarly, if the marble you take out of the box labeled BW is white.

INSCRIBING AND INSCRIBING

The picture shows an equilateral triangle inscribed in a circle inscribed in an equilateral triangle. What is the ratio between the areas of the two triangles?::

If T is the area of the big triangle, and S is the area of the small triangle, what is $\frac{T}{S}$?

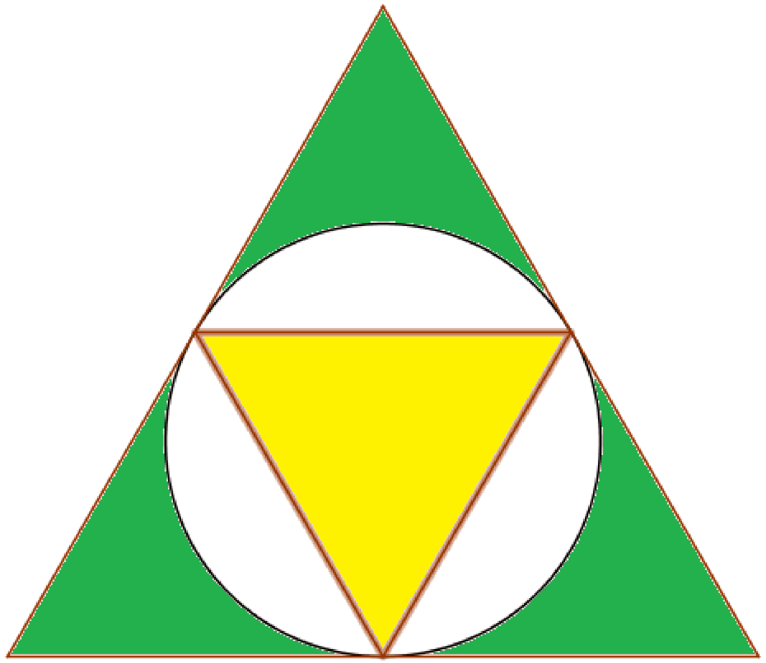
(From Martin Gardner's *Puzzle Tales*.)



INSCRIBING AND INSCRIBING - SOLUTION

The picture shows an equilateral triangle inscribed in a circle inscribed in an equilateral triangle. What is the ratio between the areas of the two triangles?::
If T is the area of the big triangle, and S is the area of the small triangle, what is $\frac{T}{S}$?
(From Martin Gardner's *Puzzle Tales*.)

$$\frac{T}{S} = 4.$$



HOW DO GARDENS GROW?

A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square.

By how many square feet does this increase the area of the garden?

(AMC 8, 1999)

HOW DO GARDENS GROW?

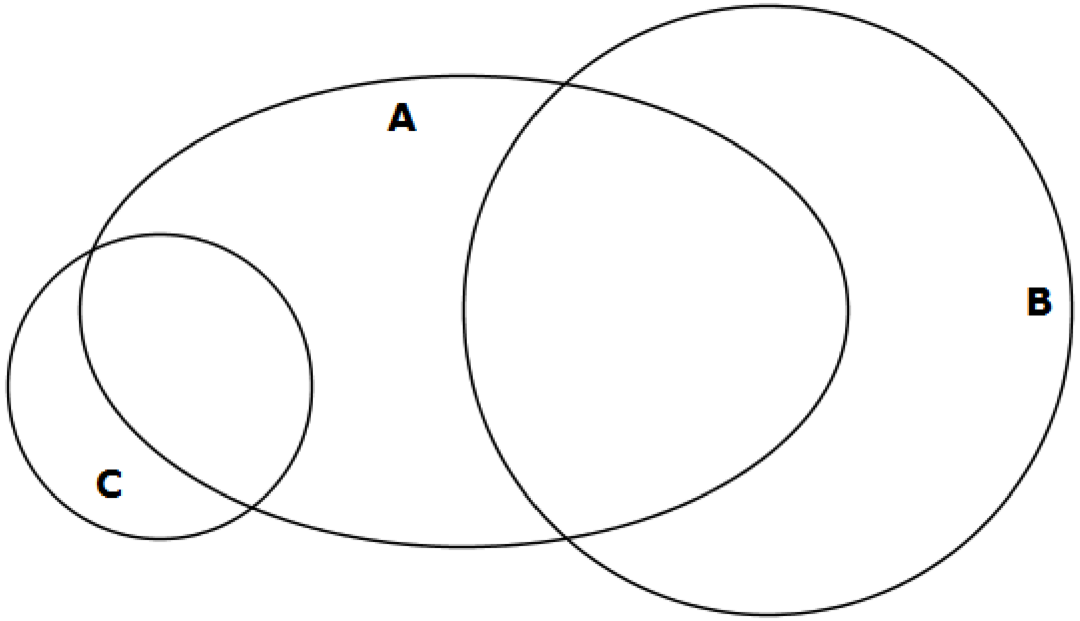
A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square.

By how many square feet does this increase the area of the garden?

(AMC 8, 1999)

SOLUTION: By 400 square feet.

MORE GARDENING



Three flower beds overlap as shown.
Bed A has 500 plants.

Bed B has 450 plants.

Bed C has 350 plants.

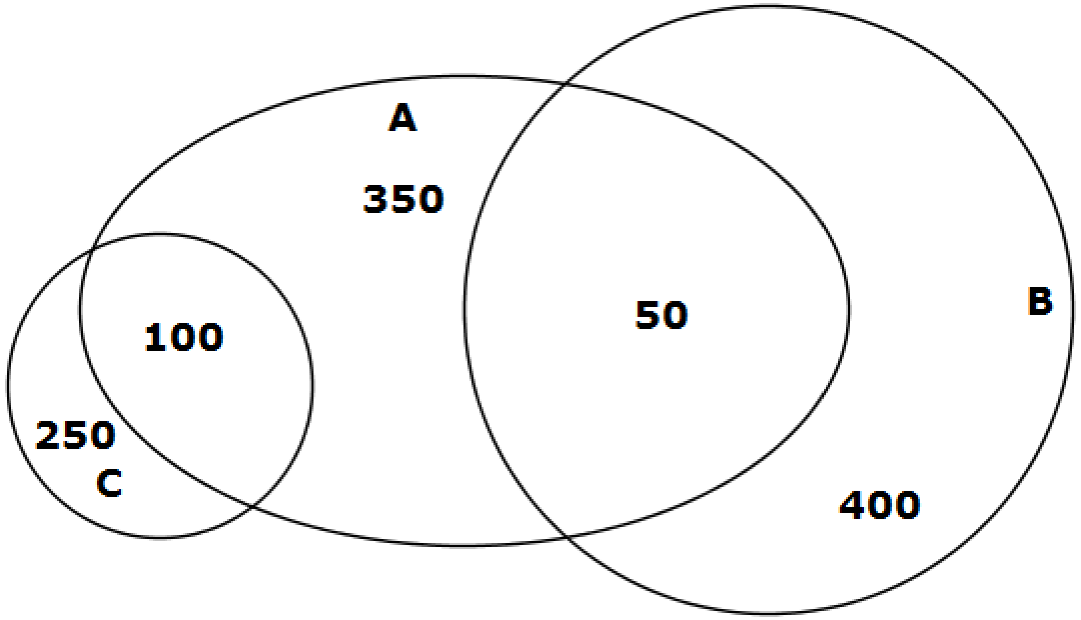
Beds A and B share 50 plants.

Beds A and C share 100 plants

What is the total number of plants?

(AMC 8, 1999)

MORE GARDENING - SOLUTION



1150 plants

Three flower beds overlap as shown.
Bed A has 500 plants.

Bed B has 450 plants.

Bed C has 350 plants.

Beds A and B share 50 plants.

Beds A and C share 100 plants

What is the total number of plants?

(AMC 8, 1999)

PAINTING PARTNERS

Peter, Paul, and Mary have to paint a 135-meter-long wall. Peter can paint three times as fast as Mary and Mary paints twice as fast as Paul. They divide the wall into three sections. Peter paints the section on one side, Paul on the other side, Mary in the middle. They all take the same time to paint their sections. How wide is each section?

PAINTING PARTNERS - SOLUTION

Peter, Paul, and Mary have to paint a 135-meter-long wall. Peter can paint three times as fast as Mary and Mary paints twice as fast as Paul. They divide the wall into three sections. Peter paints the section on one side, Paul on the other side, Mary in the middle. They all take the same time to paint their sections. How wide is each section?

Peter's section is 90 meters wide,

Mary's is 30 meters wide,

and Paul's is 15 meters wide.

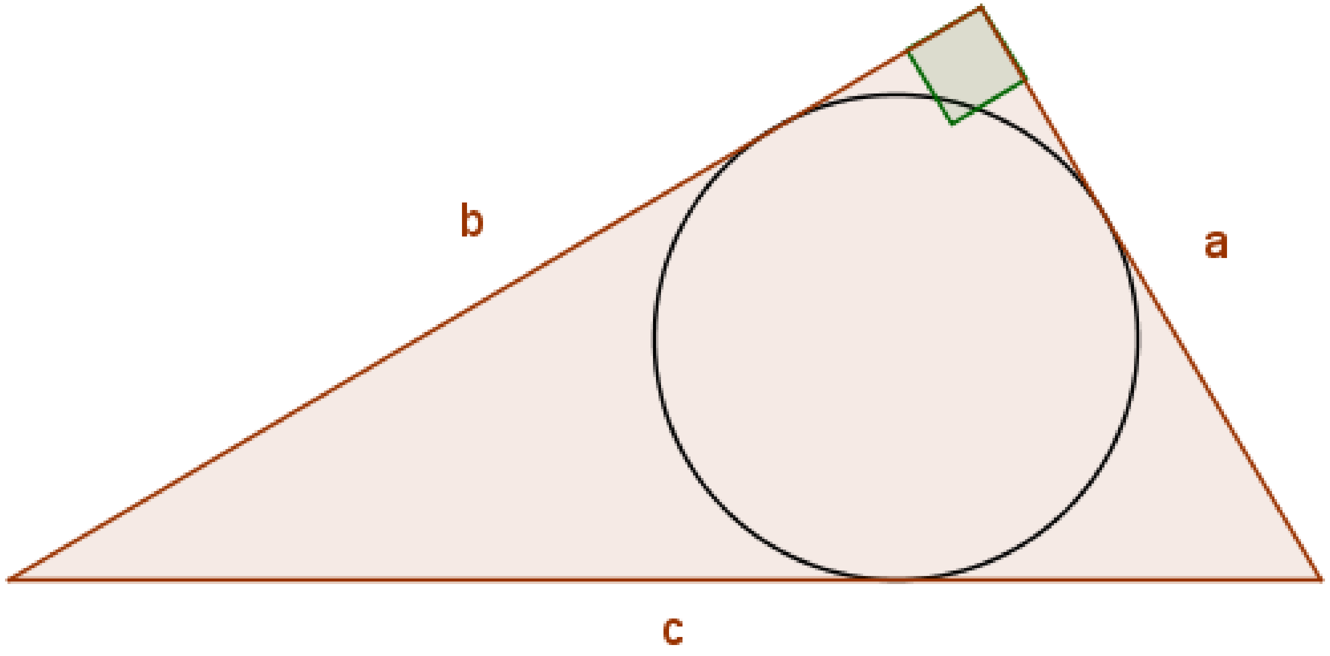
GEOMETRY IS GOOD

A circle has been inscribed in the right triangle of legs

$$a = 33, b = 56$$

and hypotenuse c.

What is the radius r of the circle?



GEOMETRY IS GOOD

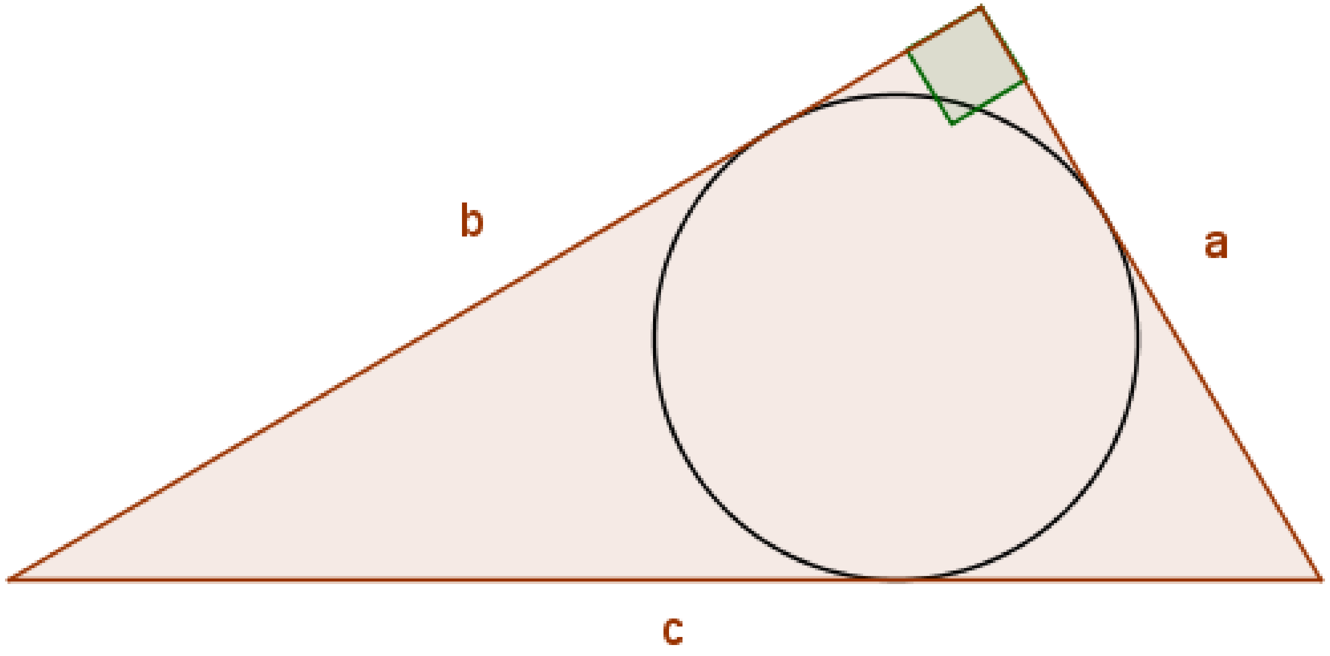
A circle has been inscribed in the right triangle of legs

$$a = 33, b = 56$$

and hypotenuse c.

What is the radius r of the circle?

Last time we learned (it is easy to see why) that the area A of a triangle satisfied $A = rs$, where s is the semi perimeter. Using Pythagoras, we wee that $c = 65$, so $s = 77$. We also see $A = 924$. Thus $r = A/s = 12$.

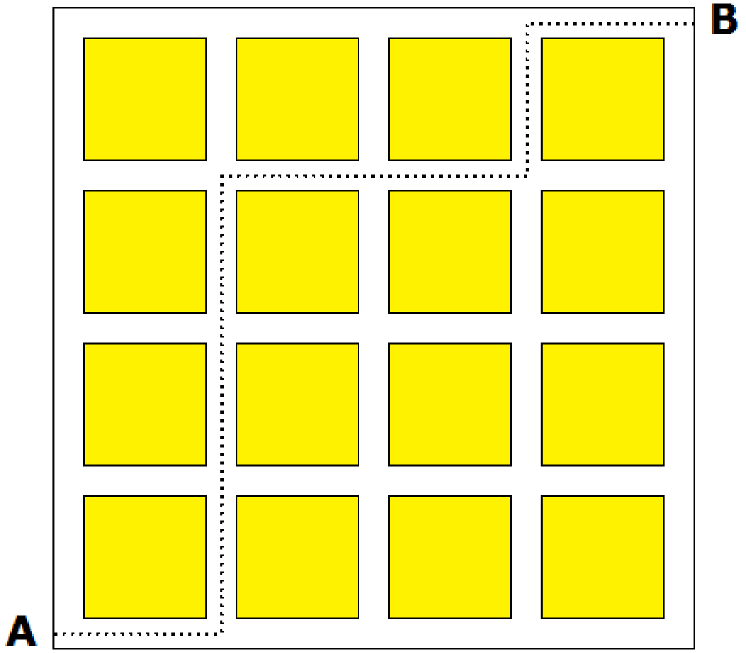


CITY TRAVEL

The picture shows a diagram of 16 blocks of a city.

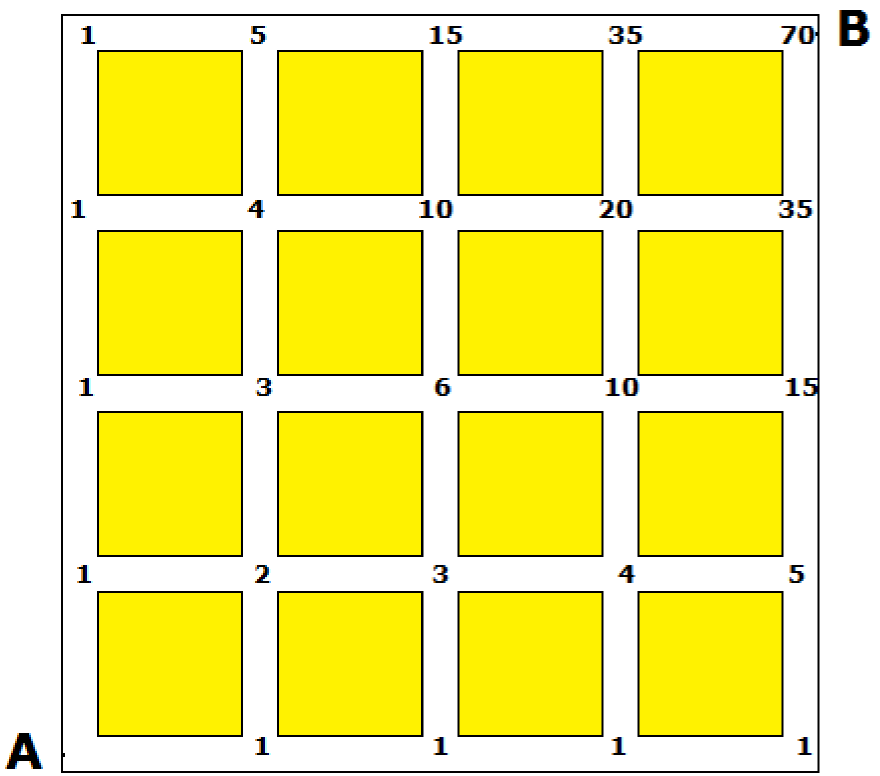
In how many ways can one travel from A to B moving only upward and to the right.

One such route is indicated by the dotted lines.



CITY TRAVEL

Solution. Starting at A figure out how many routes there are to each intersection. Since one can only move up and to the right, to every intersection on the leftmost side of the diagram, and every intersection on the bottommost side there is only one route. For any other intersection to get to it you have to have first reached the intersection one block south of it or the intersection one block west of it. If there are n routes to one block south, m routes to one block west, there will be $m + n$ routes to the intersection in question. The number of routes develops exactly like the entries in Pascal's triangle! In the picture I wrote at each intersection how many different routes lead up to it.



There are 70 different routes from A to B

COUNTING NUMBERS

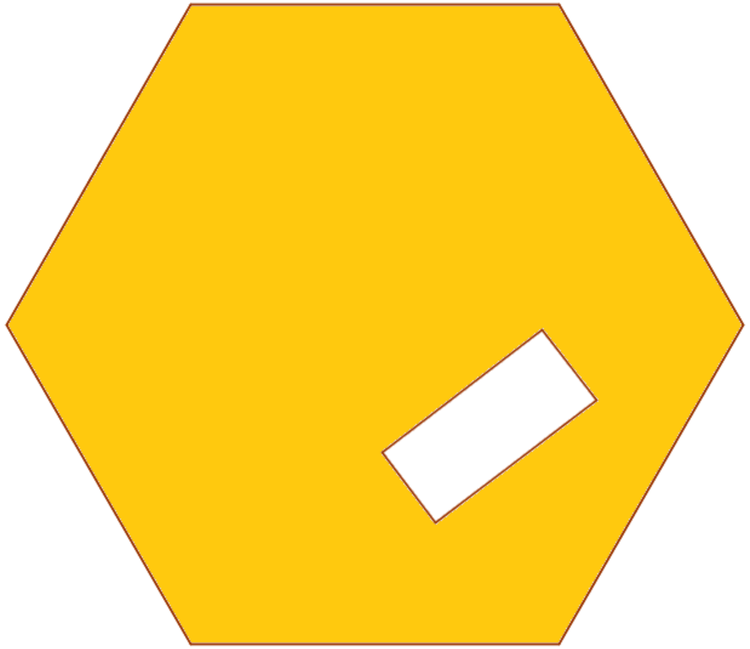
- For a class project Donatello made a list of all numbers that were larger than 100, less than 1000, and that were multiples of 3, 5 or 7. His list contained 312, 305, and 350. It did not contain 121 or 701.
- How many numbers are in Donatello's list?

COUNTING NUMBERS

- For a class project Donatello made a list of all numbers that were larger than 100, less than 1000, and that were multiples of 3, 5 or 7. His list contained 312, 305, and 350. It did not contain 121 or 701.
- How many numbers are in Donatello's list?

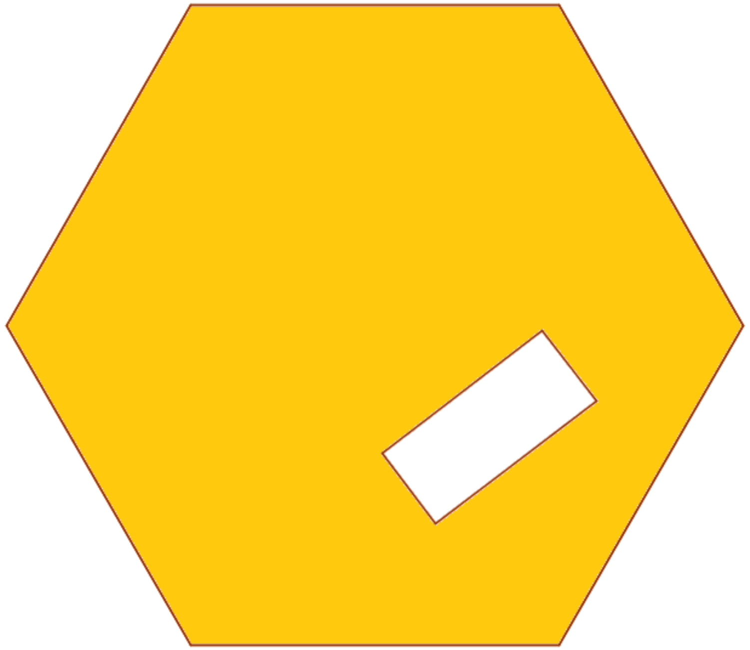
- **Solution**: 487 Numbers

CUT THE HEXAGON



- A rectangle has been cut out of a regular hexagon. Draw a straight line that divides the shaded gure (the hexagon minus the rectangle) into two parts of equal area.

CUT THE HEXAGON - SOLUTION



- A rectangle has been cut out of a regular hexagon. Draw a straight line that divides the shaded gure (the hexagon minus the rectangle) into two parts of equal area.