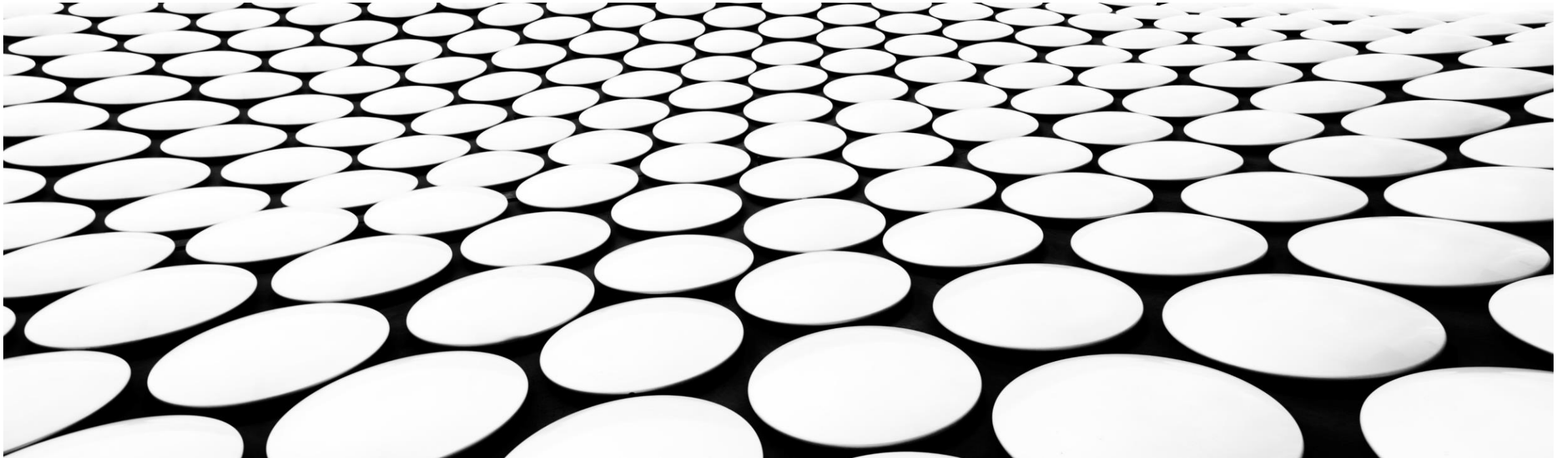# MATH CIRCLE AT FAU

MORE COUNTING, THIS AND THAT

A QUICK REFRESHER

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$$n! = 1 \times 2 \times 3 \times \cdots \times n$$
$$1! = 1, \; 2! = 2, \; 3! = 6, \; 4! = 24, \; 5! = 120, \ldots$$
$$0! = 1 \text{ (Because it works)}$$
$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!},$$
$$\binom{n}{0} = 1 \quad \binom{n}{1} = n, \quad \binom{n}{2} = \frac{n(n-1)}{2},$$
$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}, \ldots, \binom{n}{n} = 1.$$
$$\binom{n}{k} = 0 \text{ if } k > n.$$

```
              1
          1       1
       1      2      1
     1     3     3     1
   1    4     6     4    1
  1   5    10    10    5   1
 1  6   15   20   15   6   1
```

MS. NAKAMURA'S CLASS AGAIN

Ms. Nakamura's class has 25 students, 10 boys and 15 girls. Of the students, 4 boys and 7 girls are excellent singers; the rest of the students are just so-so. Ms. Nakamura has to assemble a cast for a production of an opera. She needs 2 boys and 3 girls with excellent voices for the lead roles, and then a chorus of 5 boys and 5 girls from among the remaining students, making sure that the excellent singers not chosen for the lead roles are part of the chorus. In how many different ways can such a cast be assembled?

NOW FOR SOMETHING COMPLETELY DIFFERENT

- Two people left at dawn, at the exact same time, one traveling from A to B, the other one from B to A. They travel at a constant speed, without stopping. They meet at noon. The first one arrives at B at 4 p.m., the second one arrives at A at 9 p.m.

- At what time was dawn that day?

THE 2, 3, 5 QUESTION

- How many numbers in the range 1-1000 are NOT divisible by 2, 3, or 5?

A TILING PROBLEM

- Leonardo wishes to tile a rectangular path that is 10 feet 120 inches long and 10inches wide. He wants to use 24 10″ × 5″ rectangular tiles, all of the same green color, to do this. In how many different ways can Leonardolay the tiles?

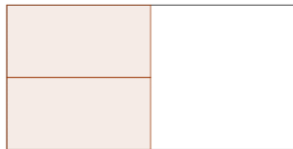# A TILING PROBLEM

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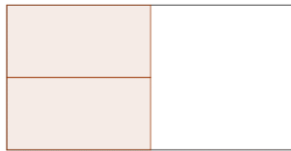
Once you figure out the answer, here are two additional challenges: What is the answer if the strip to be tiled is $15''$ wide? What if the tiles are of different colors? Say black and white. (To think about at home)