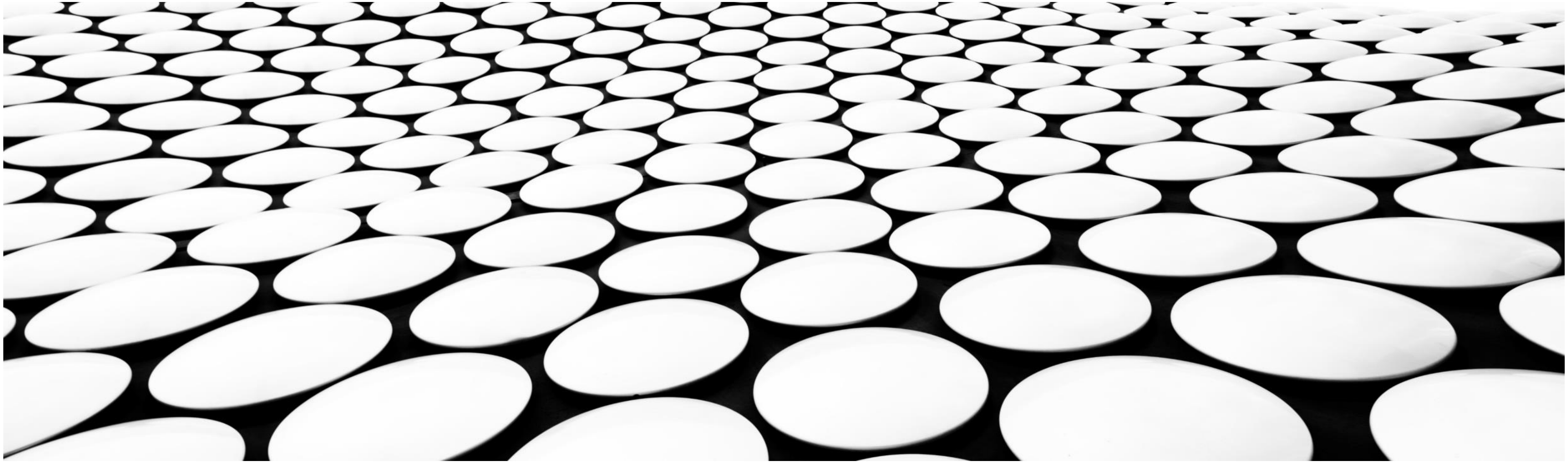
# MATH CIRCLE AT FAU

3/2/2024



# HERON'S FORMULA

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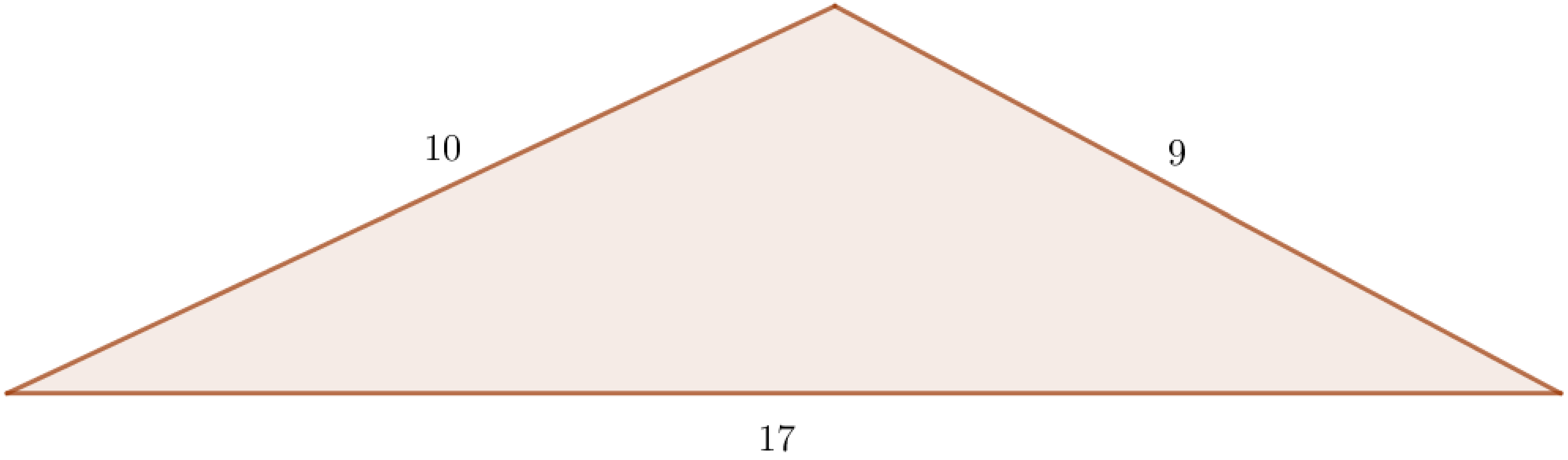
- Heron of Alexandria was a great mathematician and inventor.
- One of his discoveries is a formula for the area of a triangle.
- Do you know it? If not, you will learn it by paying attention.

# A TRIANGULAR QUESTION

- The triangle in the picture has sides of length 17, 10, and 9.
- Find its area.



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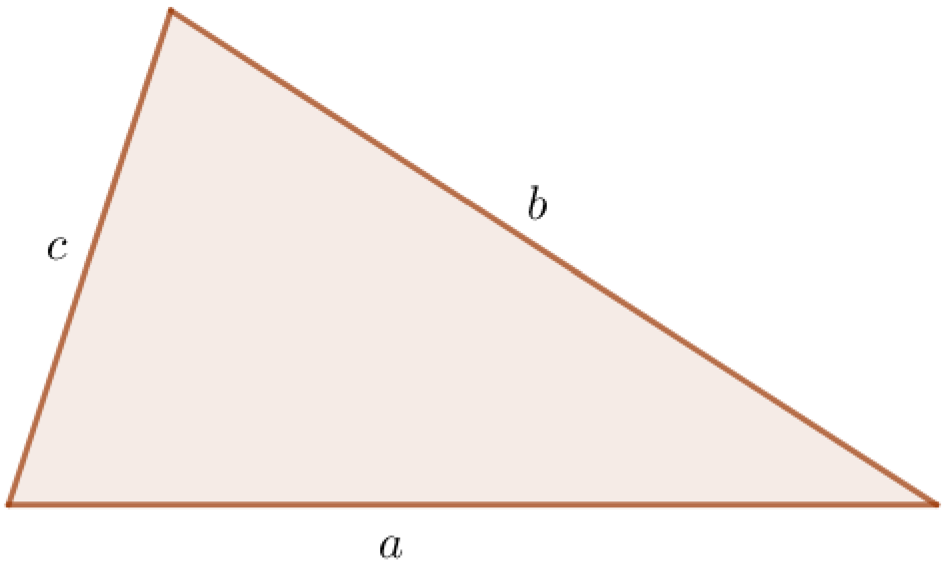


Given a triangle of sides $a, b, c$

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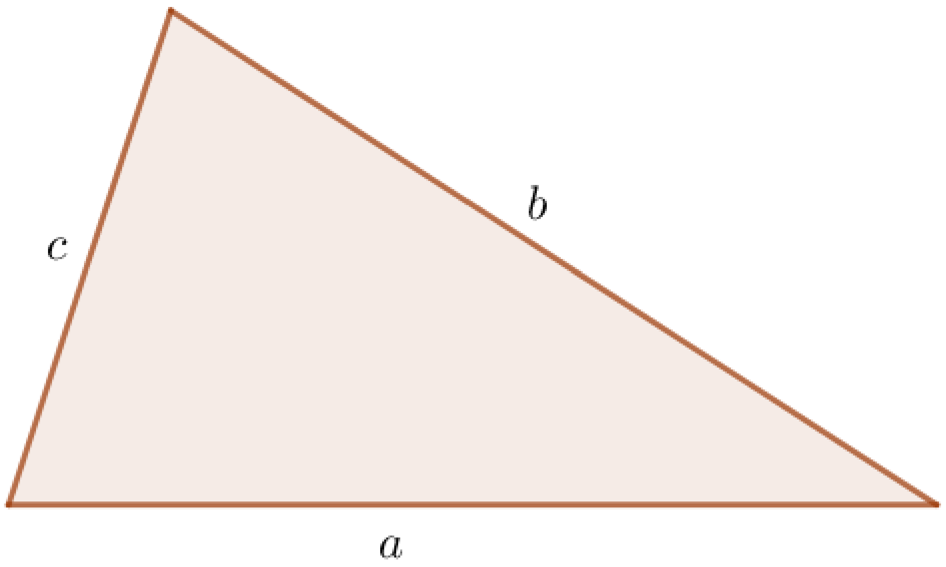


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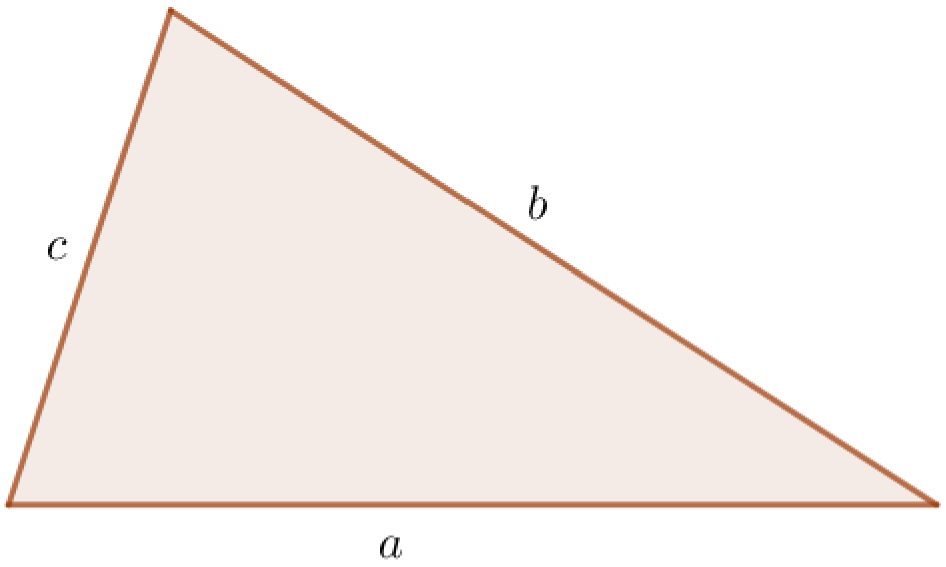


Given a triangle of sides $a, b, c$

Its semi-perimeter is $s = \frac{a+b+c}{2}$

The area is $A = \sqrt{s(s-a)(s-b)(s-c)}$

# A TRIANGULAR QUESTION

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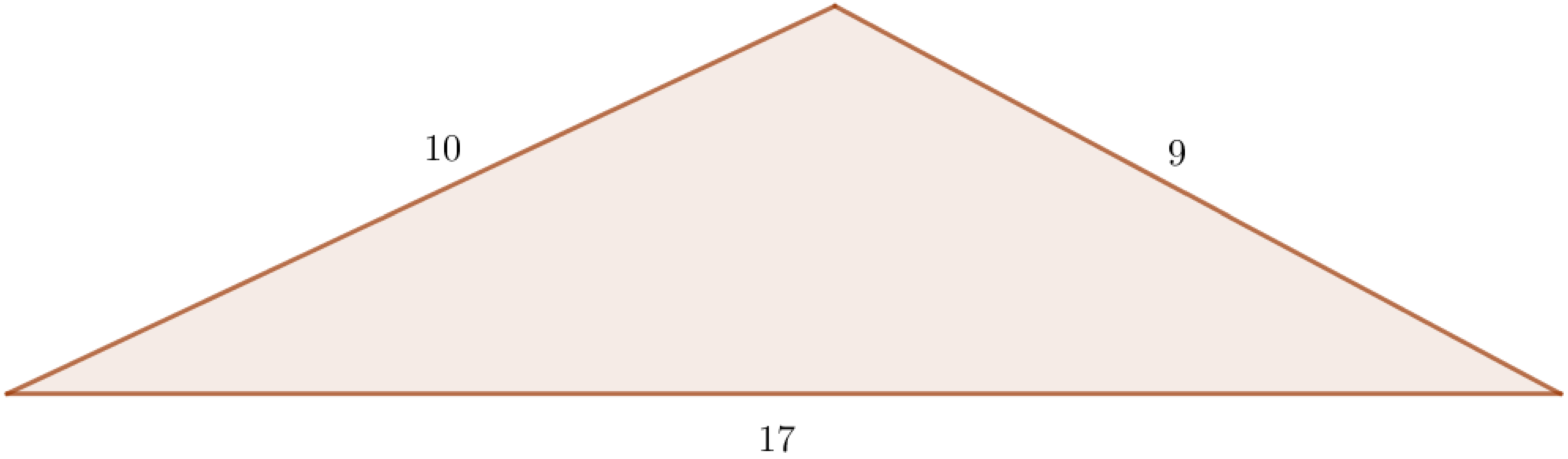

# A TRIANGULAR QUESTION

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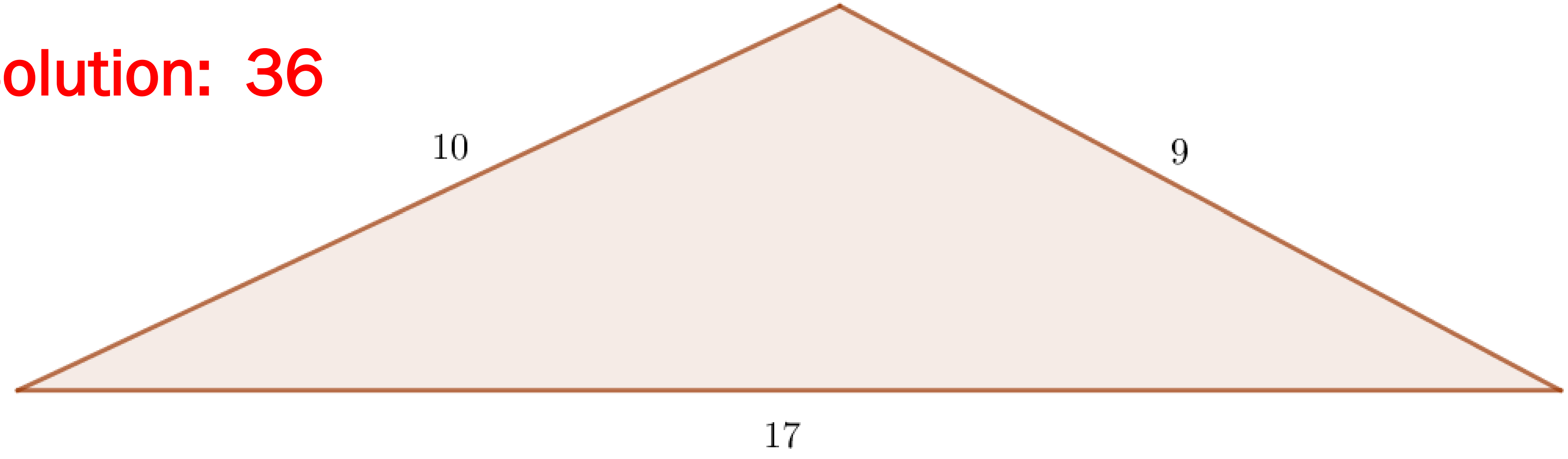
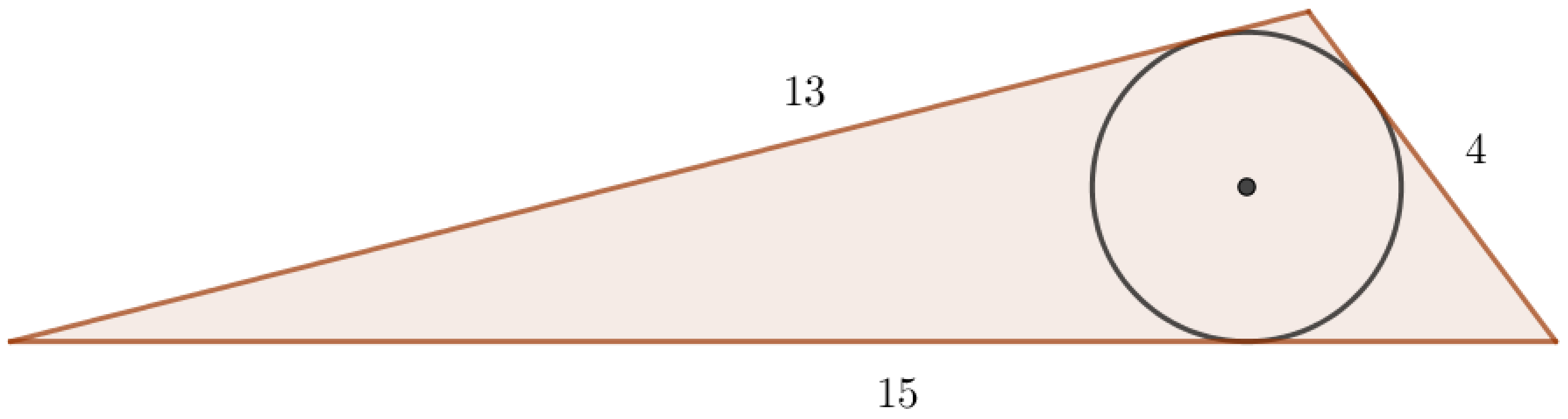
Solution: 36



# INSCRIBED CIRCLES




The pictured triangle has sides of lengths 15,13, and 4. What is the radius of the inscribed circle. The next picture may suggest something.

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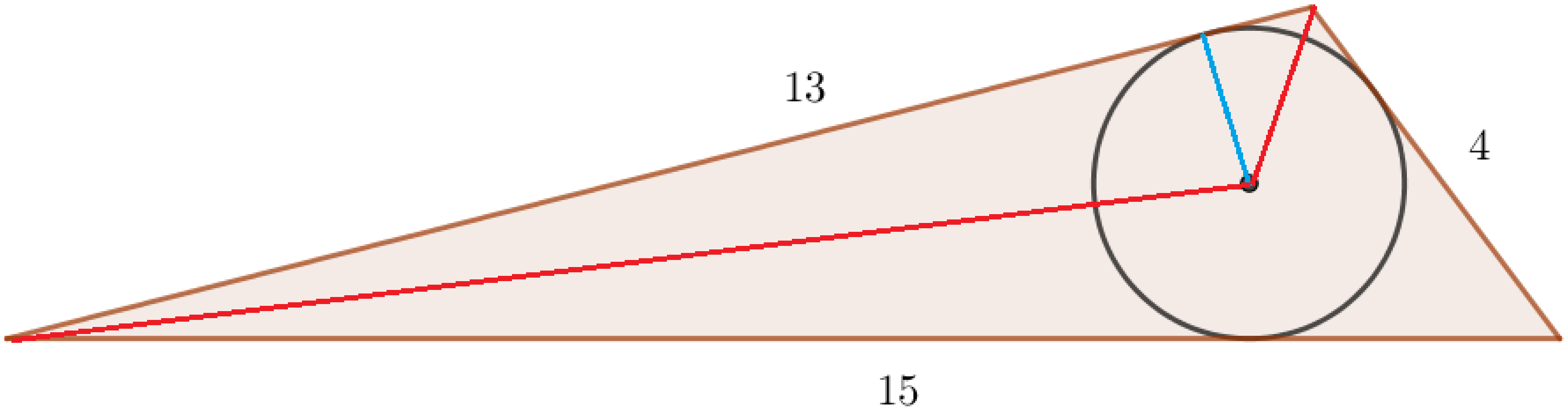


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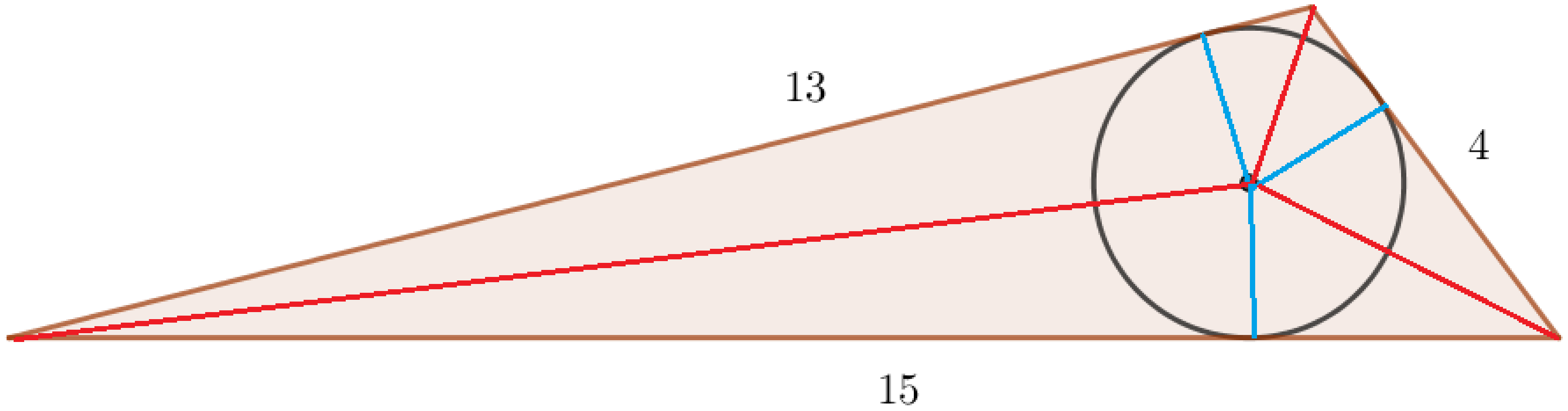
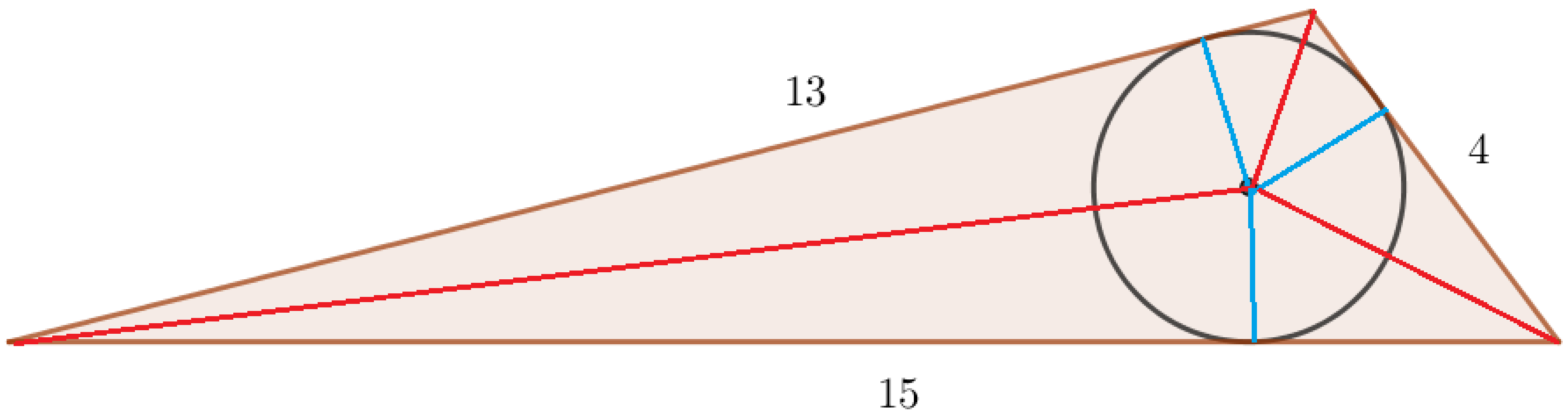
# INSCRIBED CIRCLES - SOLUTION




The red lines divide the triangle into three triangles whose base are the sides, altitudes the radius of the inscribed cricle. So, if the radius is $r$, we get the formula

$$A = sr = 16r$$

# INSCRIBED CIRCLES - SOLUTION




By Heron, $A = \sqrt{s(s-a)(s-b)(s-c)} = 24$. So $16r = 24$. We get $r = \frac{24}{16} = \frac{3}{2} = 1.5$

# MORE INSCRIBED CIRCLES

- A triangle has altitudes of lengths 24, 24, and 20.
- Explain why the triangle must be isosceles.
- Find the radius of the inscribed circle.

# MORE INSCRIBED CIRCLES - SOLUTION

- A triangle has altitudes of lengths 24, 24, and 20.
- Explain why the triangle must be isosceles.
- Find the radius of the inscribed circle.

- Let $a, b$ be the length of the sides at which the two equal altitudes of length 24 end. Then, if $A$ is the area of the triangle

$$\frac{1}{2}(24)a = A = 1/2(24)b$$

So $a = b$.

# MORE INSCRIBED CIRCLES - SOLUTION

- A triangle has altitudes of lengths 24, 24, and 20.
- Explain why the triangle must be isosceles.
- Find the radius of the inscribed circle.

- Let $c$ be the third side. From $A = \frac{1}{2}(24)a = \frac{1}{2}(20)c$ we get $c = \frac{6}{5}a$. A bit of Pythagoras shows $20^2 + \left(\frac{c}{2}\right)^2 = a^2$. We can solve to get $a = 25, c = 30$. We now calculate $A = \frac{1}{2}(20)(30) = 300$, semi-perimeter $s = \frac{1}{2}(25 + 25 +$

# POINT IN A SQUARE

- A point $P$ inside a square of side length $a$ is at the same distance $d$ from two consecutive vertices and from the side opposite the two vertices. What is $\frac{d}{a}$?