MATH CIRCLE AT FAU

2/24/2024

UPS AND DOWNS

- At the grocery store last week, small boxes of facial tissue were priced at 4 boxes for $5. This week they are on sale at 5 boxes for $4. Find the percent decrease in the price per box during the sale. (AJHSME)

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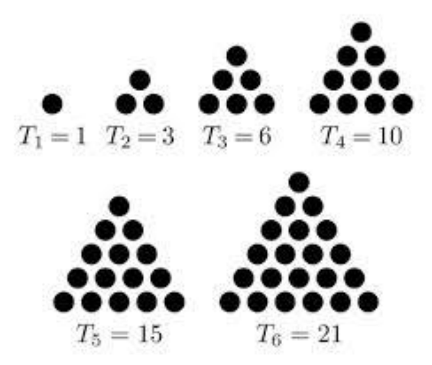
Solution. The original price per box was $ 5/4 = $ 1.25. The price goes down to $ 4/5 =$0.80, a decrease of $ 0.45 Since we are asking for the percentage of the decrease we have to figure out what percentage 0.45 is of 1.25. Now 0.45/1.25 = 0.36. The answer is 36%.

TRIANGULAR NUMBERS

- *Triangular numbers* are numbers that can be represented by triangular shapes made from dots. The first few are



- What is T_{100} ?
- Can you find an easy formula so that if I give you a number, call it n, you can tell me at once what T_n is?
- How is T_n related to $1 + 2 + \cdots + n$?

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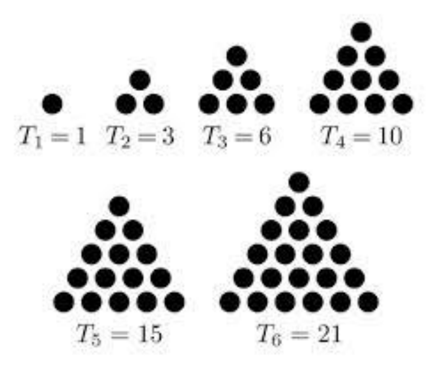
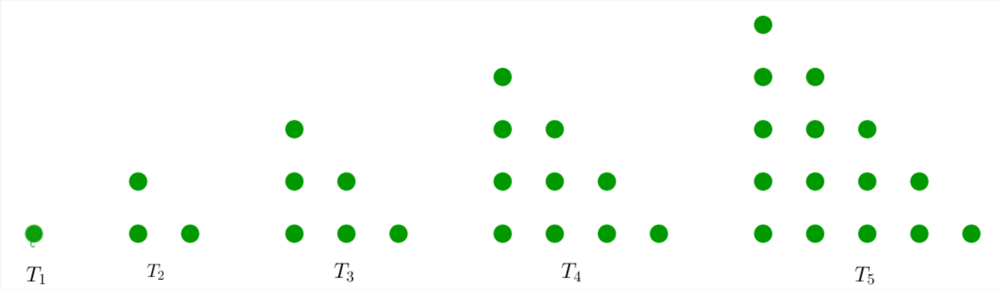


As a hint, here is another way of representing triangular numbers.



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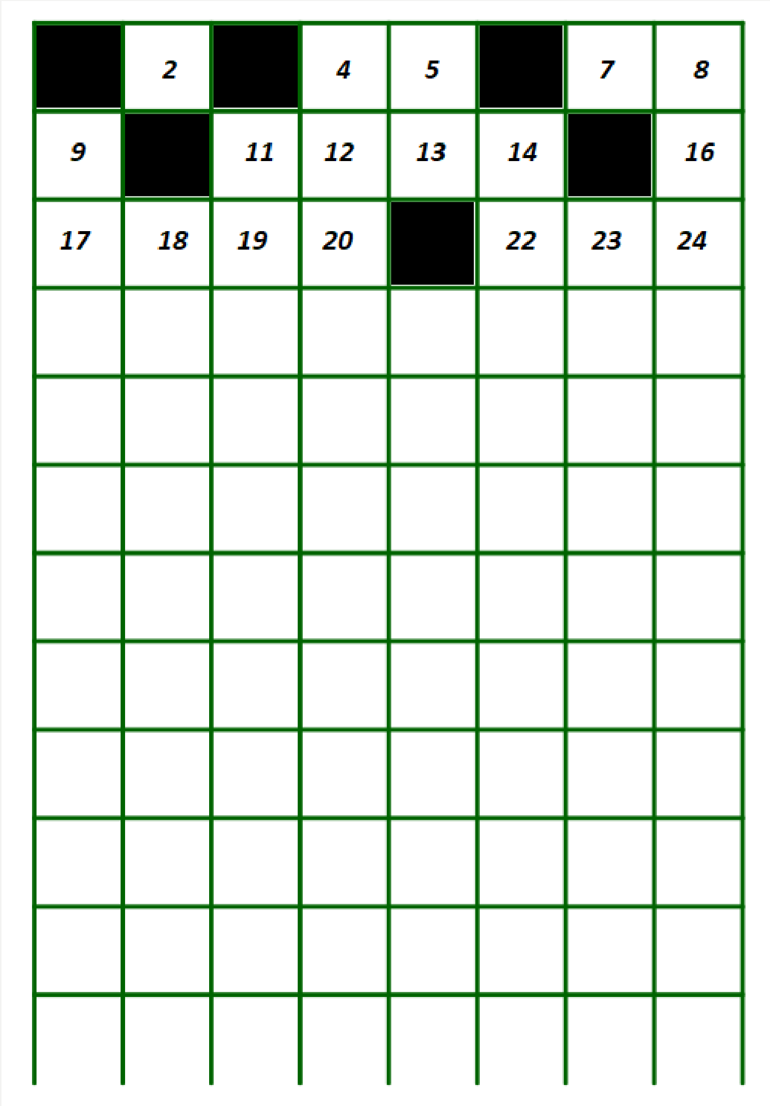
THE INFINITE BOARD

- A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skips two squares and shades square 6, skips 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column. What is the number of the shaded square that first achieves this result? (AJHSME)



ARMAND AND FRIENDS

- From before, but never solved:
- There used to be a time when people wrote letters to friends and family, instead of texting or email. In those far, far gone days, a person, lets call him Armand, wrote 7 letters to 7 friends. Armand had previously addressed 7 envelopes for the letters written. In how many ways can Armand place **every** single letter into the wrong envelope?
- (As with so many problems one should start with easy cases. For example, if there is one letter, one envelope, there are 0 ways of making a mistake. If there are two letters, lets call them $L_1$and L_2, and two envelopes $e_1$and e_2, then there is only one way; L_1 into e_2 and L_2 into e_1 .And so on.)

SOLUTION TO ARMAND'S PROBLEM

- Let s_n be the number of ways one can place n letters into n envelopes so that **no** letter is in the right envelope. We already saw that $s_1 = 0, s_2 = 1$.
- If we have n letters and n envelopes, letter n can be messed up in can go into envelopes $1, 2, \ldots, n-1$, so $n-1$ ways to mess up.. We now have two cases, (a) letter n goes into envelope k (k being one of $1, \ldots, n-1$) and letter k goes to envelope n. We are left with $n-2$ letters and their envelopes, allowing us to mess up in $s_{\{n-2\}}$ ways. Case (b), letter n goes into an envelope k, but letter k does not go into envelope n. If we now assign envelope n to letter k, we have $n-1$ letters and $n-1$ envelopes. Placing letter k into envelope n can be considered having this letter in the right place, so we have a total of s_{n-1} ways to mess up. We get the formula
$$s_n = (n-1)\left(s_{\{n-1\}} + s_{\{n-2\}}\right).$$
- $s_1 = 0, s_2 = 1, s_3 = 2(s_1 + s_2) = 2, s_4 = 3(s_2 + s_3) = 9, s_5 = 4(s_3 + s_4) = 44, s_6 = 5(s_4 + s_5) = 265$

and finally

- $s_7 = 6(s_5 + s_6) = 1854.$