

# Global Stability and Bifurcation Analysis in a Cholera model with a Poisson Process in Pathogen-host Encounter

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The endemic and re-emergent cholera outbreaks are still a great threat to both developing and developed countries. In this project, complicated cholera transmission dynamics is captured by a simple compartmental model with a novel encounter rate, which considers stochastic fluctuations in pathogen-host encounters. The transmission rates for human-to-human and environment-to-human routes are investigated for the existence of two endemic equilibriums and backward bifurcation when the basic reproduction number is less than one. The condition for a complete disease elimination is obtained from a globally stable disease free equilibrium. The condition for a persistent endemic cholera is obtained from a globally stable endemic equilibrium. A Lyapunov function and compound metrics are employed for global analyses. Hopf bifurcation occurs and provides oscillating source for semi-annual to multi-annual cholera outbreak patterns. Two-parameter bifurcation analysis and diagrams (with respect to two transmission rates for human-to-human and environment-to-human routes) illustrate the parameter regions for the array of disease dynamics. Numerical simulations are shown to demonstrate the corresponding dynamical behaviors.

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\*Mini-Symposium: Disease Modelling - from Within Host to Population