Numerical implementation of R_0 in population dynamics

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The already famous basic reproduction number R_0 plays a key role the analysis and interpretation of population dynamics models. Although the theoretical framework has been well established by many authors (see [**?**] and the references therein), there is lot of room for the efficient computation.

Non-linear ecological and epidemiological models can be linearized around the extinction equilibrium and the disease-free equilibrium, respectively. Then, for the standard case, we can get the next-generation operator from the evolution equations. The characterization of R_0 as the spectral radius of next generation operators is rather elegant, but simultaneously poses serious obstacles to its practical determination when there are no explicit formulas at hand.

In this work we address the problem from the numerical point of view, through suitable reductions of the relevant operators to matrices, thus computing the sought quantity by solving standard eigenvalue problems. We pay especial attention when the operators involved are not compact and finally we have checked the accuracy of the numerical results with the exact solution for specific cases.

References

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