Singularly perturbed non-local diffusion systems applied to disease models

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We study a model, suitable for vector-borne diseases, where we assume that the human hosts epidemiology acts on a much slower time scale than the one of mosquitoes transmitting as a vector from human to human, due to their vastly different life cycles. This particular model also includes the spatial movement of both vectors and humans, getting a system of non-local and local spatial dynamics. In a convenient setting, we prove convergence of solutions. (Joined work with M. Pereira)

The model proposed takes the following form, where i and j will model the density of infected human and vector population.

$$\begin{cases} \dfrac{\partial i}{\partial t} = \alpha_h(1-i)j - \beta_h i + d_1 K_J\, i, \\ \\ \dfrac{\partial j}{\partial t} = \dfrac{\alpha_v}{\varepsilon}(1-j)i - \dfrac{\beta_v}{\varepsilon} j + d_2 \Delta j, \end{cases} \qquad x \in \Omega,\ t > 0. \tag{1}$$

We work in a regular bounded domain $\Omega \subset \mathbb{R}^n$ with exterior unit normal N. Also, we take the homogeneous Neumann boundary condition to the function j

$$\frac{\partial j}{\partial N} = 0, \quad x \in \partial\Omega. \tag{2}$$

The constants α_h, α_v, β_h, β_v, d_1 and d_2 are positive, Δ denotes the Laplacian differential operator and K_J is the following nonlocal operator

$$K_J\, i(x) = \int_\Omega J(x-y)(i(y) - i(x))dy, \quad x \in \Omega.$$

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We assume that the kernel J satisfies the hypotheses

$$(\mathbf{H_J}) \qquad \begin{gathered} J \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}) \text{ is non-negative with} \\ J(0) > 0,\ J(-x) = J(x) \text{ for every } x \in \mathbb{R}^n, \text{ and} \\ \int_{\mathbb{R}^N} J(x)\, dx = 1. \end{gathered}$$

Under these conditions, the K_J is known as a nonlocal operator with non-singular kernel and Neumann condition.

Although the simple toy model (1) is useful to show the difficulties when one deals with non-local operators (see [1]) coupled with the laplacian, other versions using discrete networks, ODEs or fractional Laplacians have been used in applications (see [2, 3, 4, 5]).

References

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