

On a 2D avascular model of glioma with weak Allee effect

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Recent studies reveal that Allee effect may play important roles in the growth of tumor. We present one of the first mathematical models of avascular tumor that incorporates the weak Allee effect. The model considers the densities of tumor cells in three stages: proliferating cells, quiescent cells and necrotic cells. We investigate how Allee effect impacts the growth of the avascular tumor. We also investigate the effect of apoptosis of proliferating cells and necrosis of quiescent cells. The system is numerically solved in 2D using different sets of parameters. We show that Allee effect and apoptosis play important roles in the growth of tumor and the formation of necrotic core.

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